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THEORY OF NEUTRON COUNTERS USING PROTON RECOILS FROM PARAFFIN

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ABSTRACT

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General methods of calculating bias curves for paraffin detector type neutron counters are developed. Using these techniques formulas are found for several special cases, including three counters used in the laboratory. Graphs indicating the cause of various undesirable effects and how they can be reduced are included. These may be of assistance in counter design. Various approximations of use in calculating the response of more complicated counters are indicated.



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THEORY OF NEUTRON COUNTERS USING PROTON RECOILS FROM PARAFFINI. Introduction

In this report general methods for calculating the response characteristics of paraffin detector type neutron counters will be outlined and then applied to special counters. To make the methods clear many particular cases will be treated in some detail. Several of the counters calculated were actually used in the laboratory, while the results of the other cases can be used both to see how various effects can be altered by proper design and, when correctly combined, to estimate the response of a counter whose nature is too complicated to permit exact treatment. Formulas and graphs will be given for each counter considered.

By a "paraffin detector type neutron counter" is meant a counter in which a neutron beam falls on a radiator of some hydrogenous material causing proton recoils which are then counted in an ionization chamber. To be definite the hydrogenous material will always be called "paraffin" and the gas in the chamber "argon", although other substances such as glycerol tristearate and xenon may be used as detector and ionizing gas respectively.

The sensitivity of a counter is a function of the bias setting. This we express in terms of the minimum pulse size (P) that can be recorded at the given bias. For this discussion we define the sensitivity as:

$$S(P) = \frac{\text{number of pulses of magnitude greater than } P}{\text{number of recoils in the paraffin}}$$

(Note that this is not the customary definition of sensitivity. The latter is the number of pulses greater than P per neutron traversing the paraffin. It is our S(P) multiplied by the paraffin thickness and the hydrogen cross-section per unit volume.)

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The differential sensitivity (abbreviated D.S.) is given by:

$$D.S.(P) dP = \frac{\text{number of pulses of magnitude between } P \text{ and } P + dP}{\text{number of recoils in the paraffin}}$$

It is readily seen that: $D.S. = \left| \frac{dS}{dP} \right|$.

The problem for a given counter will be to calculate the two quantities $S(P)$ and $D.S.(P)$. In particular it is desirable to know their dependence on electron collection, paraffin thickness, wall effects, the proton ranges, and the geometry of the ion chamber.

Although the method to be outlined holds in general, it will be applied with the following simplifications (which are usually quite valid).

- a) The incident neutron beam is monochromatic and perpendicular to the paraffin surface.
- b) Variation in paraffin thickness from point to point on the surface is negligible.
- c) No appreciable attenuation of the neutron beam is caused by the paraffin.
- d) Edge effects due to variation of the electric field in the chamber are eliminated by guard rings.

Moreover, in all but one case a range-energy relation of the form $R \sim E^{3/2}$ will be used. The exception occurs in one counter where a paraffin range $R = \alpha E^{3/2} + \beta E$ was assumed.

II. General Theory

The first step is to find the expression for the pulse (P) (i.e., the change in potential difference between the chamber electrodes) produced by a recoil proton passing through the chamber. P will in general depend on n coordinates (q_1, q_2, \dots, q_n) describing the given proton. These coordinates usually specify such things as the angle at which the proton enters the chamber and the position in the paraffin where the recoil originates. For simplicity we will first consider the case where P depends

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on only one coordinate (q). Express the number of protons having values of this coordinate between q and $q + dq$. This will be of the form $N(q)dq$. Now let the number of protons producing a pulse between P and $P + dP$ be $L(P)dP$. Since this must be the same as the number of recoils between q and $q + dq$ we have:

$$L(P)dP = N(q)dq$$

Thus: $L(P) = N(q) dq/dP$. In this formula we must regard q as a function of P given by the pulse size formula. More exactly the relation is:

$$L(P) = N(q(P)) \left| \frac{dq}{dP} \right|,$$

since negative numbers of protons or of pulses have no significance here. If M is the total number of recoils having all values of q , the differential sensitivity is given by:

$$D.S.(P) = L(P)/M = (1/M) N(q(P)) \left| \frac{dq}{dP} \right|$$

and the total sensitivity is:

$$S = \int_P^{P_{\max}} D.S.(P) dP$$

In the case more than one coordinate is necessary to describe the pulse the following treatment can be used. After obtaining a pulse-size formula which says:

$$P = f(q_1, q_2, \dots, q_n)$$

the number of protons having coordinates between q_1 and $q_1 + dq_1$, q_2 and $q_2 + dq_2$, ..., q_n and $q_n + dq_n$ is found. (For brevity we will in the future describe this by saying "in the region q_1, q_2, \dots, q_n ".) This will be of the form:

$$N(q_1, q_2, \dots, q_n) dq_1 dq_2 \dots dq_n.$$

By use of the pulse size formula, P is introduced as an independent variable instead of one of the coordinates q_r . The number of protons in the region $q_1, q_2, \dots, q_{r-1}, q_{r+1}, \dots, q_n$ and producing a pulse between P and $P + dP$ is obtained as:

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$$N(q_1, q_2, \dots, q_{r-1}, P, q_{r+1}, \dots, q_n) dq_1 dq_2 \dots dq_{r-1} \left| \frac{\partial q_r}{\partial P} \right|_{q_1, q_2, \dots} dP dq_{r+1} \dots dq_n$$

The differential sensitivity is then:

$$D.S.(P) = \frac{1}{M} \int^{n-1} N(q_1, q_2, \dots, q_{r-1}, P, q_{r+1}) dq_1 dq_2 \dots dq_{r-1} \left| \frac{\partial q_r}{\partial P} \right|_{q_1, q_2, \dots} dq_{r+1} \dots dq_n$$

where M is the total number of proton recoils and the $n-1$ fold integration is to be extended over all values of the coordinates compatible with the given pulse P . (It is, of course, very important which coordinate q_r is eliminated, since some choices will considerably simplify the work. The proper choice depends entirely on the individual counter being considered and so no general rule can be given.) Total sensitivity is again obtained from:

$$S(P) = \int_P^{P_{\max}} D.S.(P) dP$$

Before applying the above let us define some terms:

- a) Thin Paraffin - This means that the energy lost by recoil protons in traversing the paraffin is negligible.
- b) Electron Collection - Only electrons are collected in the chamber and this is done in a time during which the positive ions move but a negligible distance.
- c) Positive Ion Collection - Both electrons and positive ions are collected.
- d) Parallel Plate Counter - The ion chamber consists of 2 parallel plates kept at different potentials with the paraffin flat against one plate. Electrons are collected at the opposite plate.
- e) Cylindrical Counter - A collecting wire along the axis of a cylindrical chamber collects electrons produced by proton recoils from a paraffin disc supported against the side of the cylinder. A line perpendicular to the paraffin surface through its center passes through the collecting wire.



III. Plane Parallel Counter, Thin Paraffin, Positive Ion Collection

This somewhat trivial case deserves consideration first because it most clearly illustrates the general method, and second because it gives us a standard case with which all other counters can be compared. The accompanying diagram illustrates the coordinate system that will be used for all plane parallel counters.

Let E_0 = energy of incident neutrons

a = chamber thickness

t = paraffin thickness

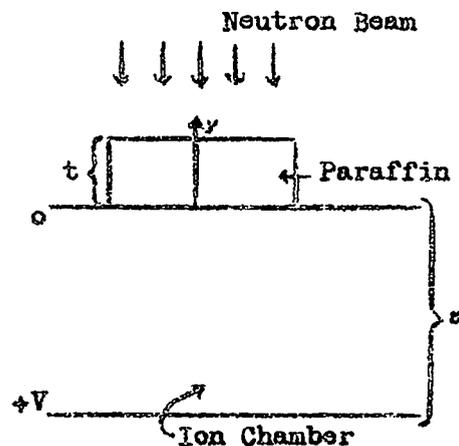
x varies from 0 at the chamber edge to t at the top edge of the paraffin

C = capacity of the chamber

R_0 = range in paraffin of protons of energy E_0

R_0' = range in argon of protons of energy E_0

$\mu = \cos \theta$, where θ is the angle between the path of the recoil proton and the direction of the neutron beam.



The upper plate of the chamber is assumed at zero potential and the bottom at + V.

a) Derivation of pulse formula

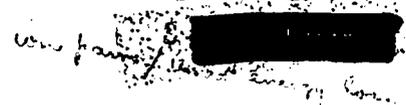
If an ion pair is produced a distance d from the bottom plate the work done on the electron in moving it to the collecting electrode is: eVd/a and that done on the positive ion is: $eV(a-d)/a$, making the total work on the ion pair eV . As this is done at the expense of the energy of the chamber considered as a condenser we have:

$$\Delta\left(\frac{1}{2} CV^2\right) = eV = CV \Delta V$$

or

$$\Delta V = e/C$$

Now consider a proton originating at x and going off at an angle $\cos^{-1} \mu$. It is easily proved that its energy is $E_0 \mu^2$. Since the paraffin is thin no energy is lost before it enters the chamber. If w is the average energy necessary to produce



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one ion pair, the total number of ion pairs formed in the chamber is wE_2 . Since each ion pair gives a pulse of $\Delta V = e/C$, the total pulse due to the proton is:

$$P = \frac{ew E_2}{C}$$

where E_2 is the energy with which the proton entered the chamber. Here it is assumed that the proton is stopped in the argon. With $E_2 = E_0 \mu^2$ we have $P = (ew/C)E_0 \mu^2$. For simplicity in parallel plate counters we will measure pulses in units of ew/C . Thus $P = E_0 \mu^2$.

b) To obtain the number of recoils in the region $q_1, q_2 \dots q_n$

The number of recoils in a volume element of unit area and thickness dx is $nv \sigma_H dx$ where σ_H is the hydrogen scattering cross section per unit volume and nv is the incident neutron flux. From the fact that the number of recoils from the isotropic n,p scattering between E and $E + dE$ is dE/E_0 , which is $d\mu^2$ from the above relation $E = E_0 \mu^2$, one has the result that the number of recoils in the region x, μ is $nv \sigma_H 2\mu d\mu dx$. Using P instead of μ as an independent variable it follows that the number of recoils in region x, P is $nv \sigma_H dx (\partial \mu^2 / \partial P)_x dx$. But $(\partial \mu^2 / \partial P)_x = 1/E_0$. Hence:

$$D.S.(P) = \frac{nv \sigma_H}{ME_0} \int_0^t dx$$

(Here x can be integrated over the whole range from 0 to t , since all values are compatible with a given P .)

$$M = (\text{total number of recoils with all values of } x) = nv \sigma_H t$$

$$\therefore D.S. = 1/E_0$$

$$S = \int_P^{P_{\max}} D.S.(P) dP = (E_0 - P)/P$$

Fig. 1 gives the graph of these two simple functions.

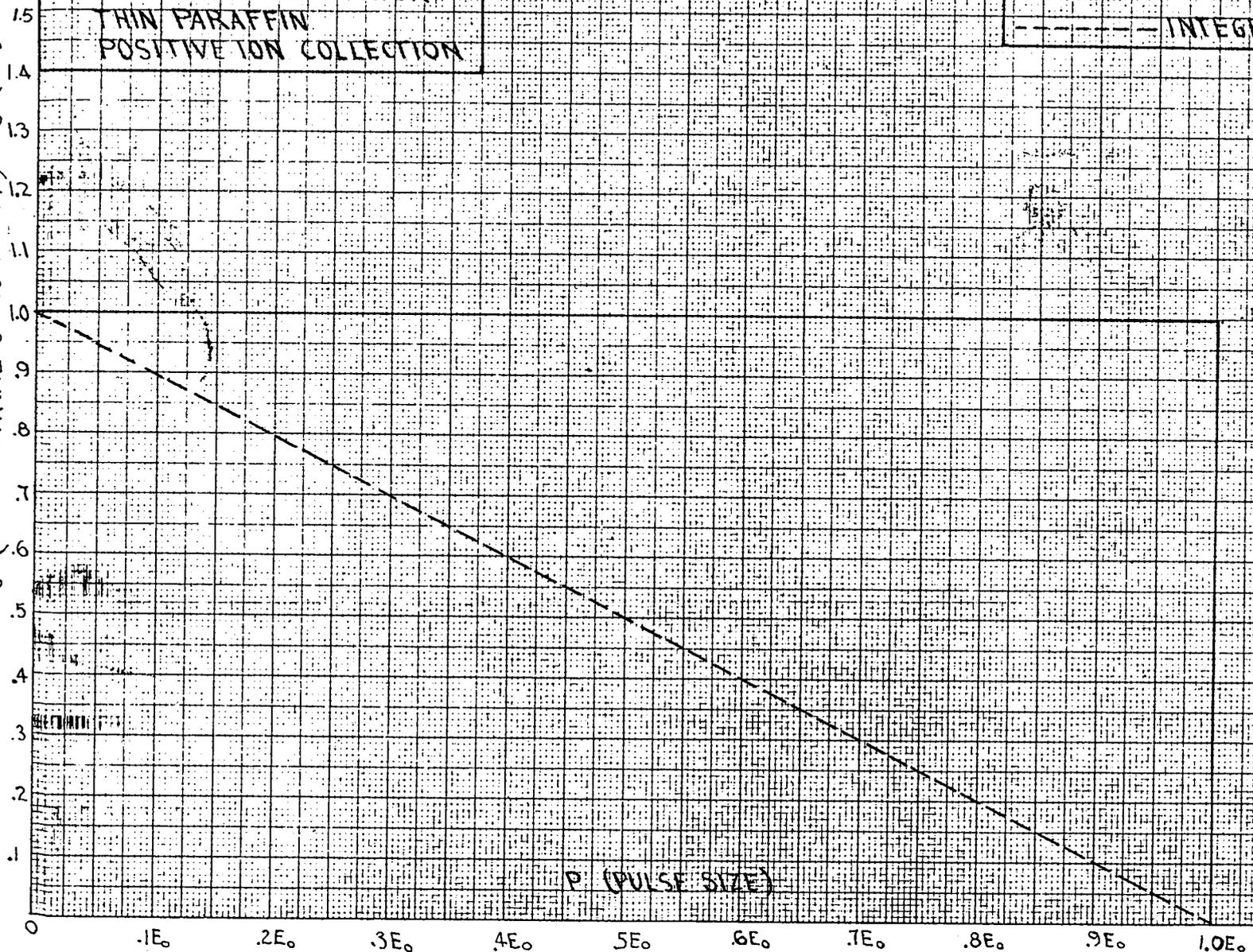
FIGURE 1
DIFFERENTIAL AND INTEGRAL SENSITIVITY CURVES

CONDITIONS:

PLANE PARALLEL COUNTER
THIN PARAFFIN
POSITIVE ION COLLECTION

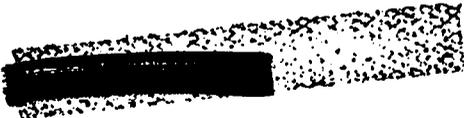
— DIFFERENTIAL SENSITIVITY
- - - INTEGRAL SENSITIVITY

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 $E_0 \times 10^7$ DIFFERENTIAL SENSITIVITY $(D.S.) = E_0 \times 10^7$ (D.S.)



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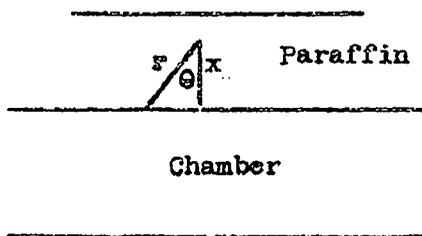


IV. Plane Parallel Counter, Thick Paraffin, Positive Ion Collection

In Section III it was found that if positive ion collection is used the pulse due to a particle entering the ion chamber with energy E_2 is (in our unit):

$$P = E_2$$

Let us express E_2 in terms of x and μ .



In paraffin the range-energy relation assumed is:

$$R = \alpha E^{3/2}; \quad R_0 = \alpha E_0^{3/2}$$

Denoting the recoil energy by E_1 ($E_1 = E_0 \mu^2$) we get:

$$R = \alpha E_1^{3/2}; \quad R-r = \alpha E_2^{3/2}; \quad r = \alpha(E_1^{3/2} - E_2^{3/2})$$

Then

$$r = x/\mu = \alpha(E_0^{3/2} \mu^3 - E_2^{3/2})$$

The number of recoils in the region x, μ is $nv \sigma_H dx d\mu^2$. Instead of x we now introduce P as a coordinate. The number of recoils in region P, μ is:

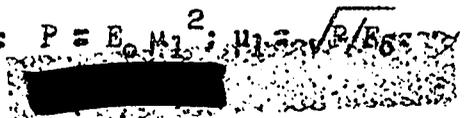
$nv \sigma_H \left| \frac{\partial x}{\partial P} \right|_{\mu} d\mu^2 dP$. Differentiating the formula for x gives:

$$\left| \frac{\partial x}{\partial E_2} \right|_{\mu} = \frac{3}{2} \alpha \mu E_2^{1/2}$$

But since $P = E_2$, $\left| \frac{\partial x}{\partial P} \right|_{\mu} = \frac{3}{2} \alpha \mu P^{1/2}$ and thus the number of recoils in region μ, P is $\frac{3}{2} nv \sigma_H \alpha \mu P^{1/2} dP d\mu^2$. Hence

$$D.S. = \frac{3}{2} \frac{nv \sigma_H}{M} \int_{\mu_1}^{\mu_2} \alpha \mu P^{1/2} d\mu^2$$

where $M =$ (total number of recoils with all E_2 and μ) = (number with all x and μ) = $nv \sigma_H t$. $\mu_1 =$ minimum μ that will yield the given pulse. It is obviously the μ corresponding to $x = 0$ that gives the given P . Thus: $P = E_0 \mu_1^2; \mu_1 = \sqrt{P/E_0}$



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Similarly $\mu_2 =$ maximum μ that yields the given pulse. Either it is the μ at $x = t$ that causes the given P or, if such μ would be greater than 1, μ_2 should be taken as 1. Expressing this mathematically we have: μ_2 is the smaller of 1 and the solution of $\mu_2^3 - t/R_0 \mu_2 - (P/E_0)^{3/2} = 0$. Using these facts one finds:

$$D.S.(P) = (R_0/E_0 t)(P/E_0)^{1/2} \left[\mu_2^3 - (P/E_0)^{3/2} \right]$$

Here μ_2 occurs as an awkward parameter. It complicates matters by preventing us from obtaining an analytic expression for $S(P)$. However, this is no great loss in any individual case, since having $D.S.(P)$ vs P either graphically or as a table of values, one can find $S(P)$ by numerical integration. Graphs of $D.S.(P)$ and $S(P)$ for several values of R_0/t are given in Figs. 2 and 3. It is seen that thick paraffin tends to cut down the counters' response to the smaller pulses. This merely says that some of the low energy recoils are absorbed in the paraffin and never reach the ion chamber. The small effect of thick paraffin on $D.S.(P)$ for large P allows us to make the following approximation for more complicated counters (if R_0/t is large). For low-energy pulses we assume thick paraffin, but ignore other effects, such as electron collection and wall effects which are unimportant at those energies. High energy pulses are then calculated taking into account the ignored effects but assuming thin paraffin. Of course if R_0/t is not large this assumption is bad as Fig. 2 shows, since the high energy response is also cut down. Prevention by the paraffin of any but those protons starting quite near the paraffin surface from producing the maximum pulse causes this effect.

FIGURE 2
DIFFERENTIAL SENSITIVITY

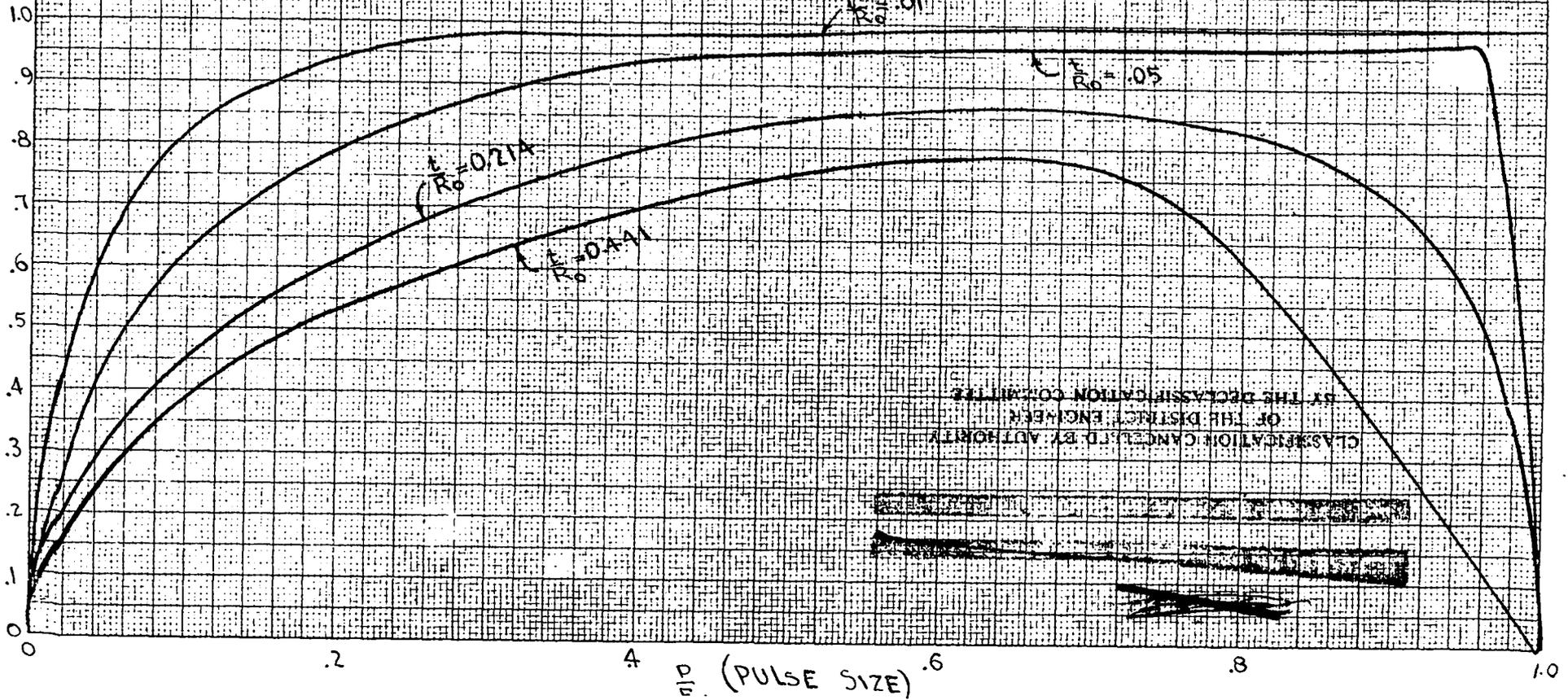
CONDITIONS:

PLANE PARALLEL COUNTER
THICK PARAFFIN
POSITIVE ION COLLECTION

L = PARAFFIN THICKNESS

R_0 = RANGE OF PROTON OF ENERGY E_0 IN PARAFFIN

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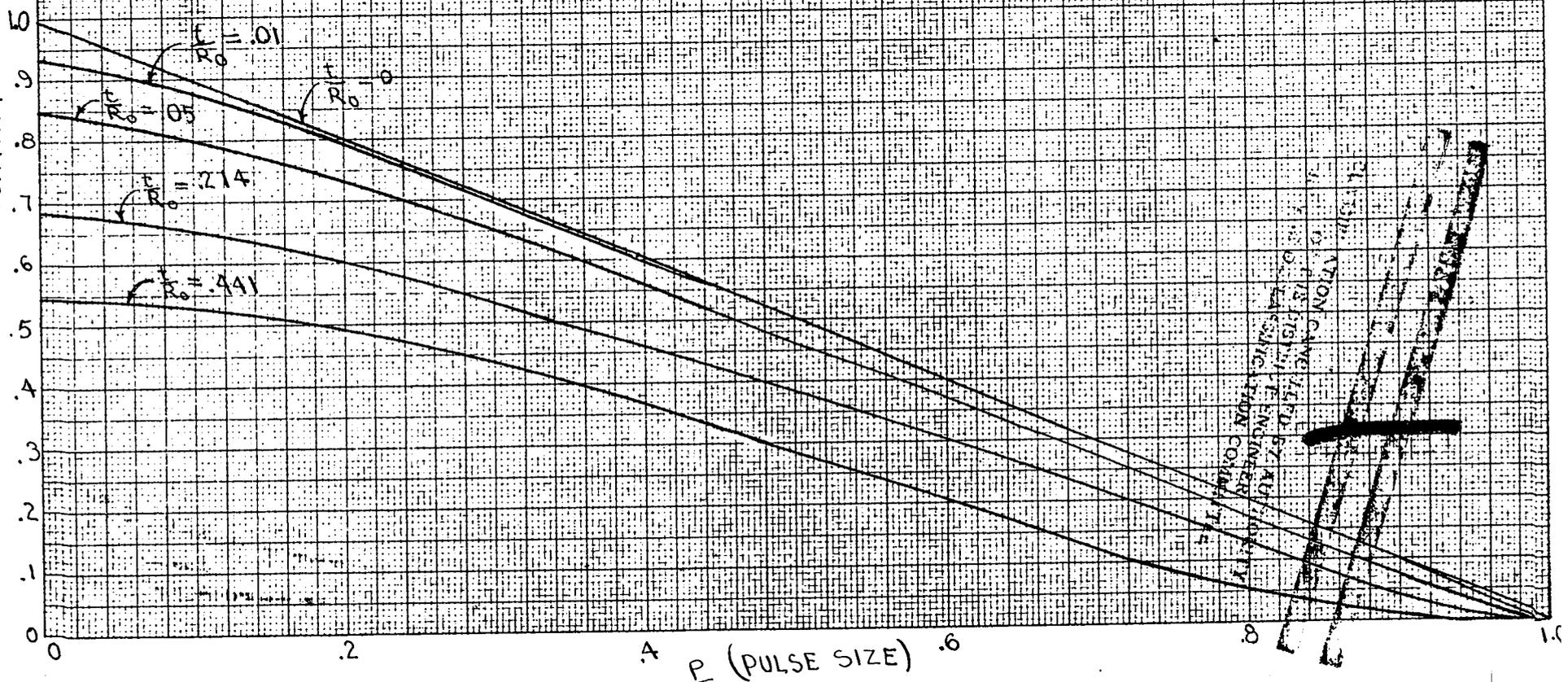
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INTEGRAL SENSITIVITY CURVES

CONDITIONS:
PLANE PARALLEL COUNTER
THICK PARAFFIN
POSITIVE ION COLLECTION

t = PARAFFIN THICKNESS
 R_0 = RANGE OF PROTONS OF
ENERGY E_0 IN PARAFFIN



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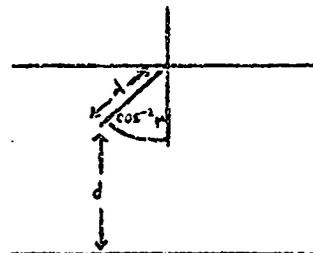
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V. Plane Parallel Counter, Thin Paraffin, Electron Collection

Electron collection forces us to use a different pulse size formula. Now the pulse caused by the formation of a single ion pair a distance d from the positive electrode is only that due to the electron (i.e., $\Delta V = ed/Ca$ instead of e/C). Consider a proton of energy E_2 entering the chamber at an angle $\cos^{-1} \mu$ with the normal. If λ measures the distance along the path, the number of ion

pairs formed in $d\lambda$ is: $-w(dE/d\lambda) d\lambda$. Now $d = a - \lambda\mu$. Thus $P = - (ew/Ca) \int_0^{R_2} (dE/d\lambda)(a - \lambda\mu) d\lambda$ where the range in argon is $R = \alpha' E^{3/2}$ and $R_2 = \alpha' E_2^{3/2}$. After expressing λ in terms of E and again setting $ew/C = 1$, integration yields;



$$P = E_2 \left(1 - \frac{3}{5} \frac{R_2}{a} \mu \right)$$

For thin paraffin the energy of a recoil entering the chamber at μ is $E_0 \mu^2$. Hence $P = E_0 \mu^2 \left(1 - \frac{3}{5} \frac{R_0'}{a} \mu^4 \right)$. The fraction of recoils in region μ is $d\mu^2$. Introducing P instead of μ as a variable gives:

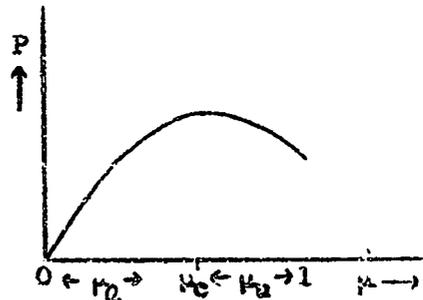
$$D.S.(P) = \frac{\partial \mu^2}{\partial P} = \frac{1}{\partial P / \partial \mu^2} = \frac{1}{E_0 \left(1 - (9/5) (R_0'/a) \mu^4 \right)}$$

where μ is obtained from the pulse size formula. Again the result is expressed in terms of a parameter. Hence $S(P)$ must be found by numerical integration. Figs. 4 and 5 show a typical example. Electron collection has the effect of pushing the maximum pulse down somewhat and causing a peaking of the differential sensitivity in the region of large pulses.

The formulas for this case are particularly useful in conjunction with those of Section IV, since to a certain approximation a plane parallel counter with thick paraffin and electron collection can be treated by using Section IV formulas for small pulses and this section's formula for large pulses.

When designing fast neutron counters (say of the order of 5 Mev) these formulas can be of considerable aid. For such energies the main worry is the peaking effect caused by electron collection. To obtain quick estimates of the differential sensitivity curves and how they depend on chamber depth and argon pressure, one can ignore the paraffin thickness and use the last expression above for D.S.(P). Fig. 6 shows how the differential sensitivity curves vary with R_0'/a .

On examining the formula it appears that even for some R_0'/a less than 1 a negative differential sensitivity will appear. If this happened for $R_0'/a > 1$, it would not be surprising since protons would not spend all their energy in the chamber and so some correction would be expected. The present situation must be explained differently. The answer lies in the vanishing of $\partial P / \partial \mu^2$ because of the competition between the low energies of wide-angle recoils and the shorter electron paths from forward recoils. P as a function of μ^2 looks roughly like the accompanying diagram. Thus some pulses can be caused by two different μ 's. The remedy is to break μ into two ranges $0 \leq \mu_l \leq \mu_c$; $\mu_c \leq \mu_u \leq 1$ with $\mu_c = \left(\frac{5}{9} \frac{a}{R_0'}\right)^{\frac{1}{4}}$. (If μ_c turns out greater than 1 this phenomenon does not take place.) Then:



$$D.S.(P) = \left| \frac{\partial \mu_l^2}{\partial P} \right| + \left| \frac{\partial \mu_u^2}{\partial P} \right|$$

$$\left| \frac{\partial \mu_k^2}{\partial P} \right| = \left| \frac{1}{E_0 \left(1 - \left(\frac{9}{5}\right) \left(\frac{R_0'}{a}\right) \mu_k^4\right)} \right|$$

with μ_k the solution of

$$P = E_0 \mu_k^2 \left(1 - \frac{3}{5} \frac{R_0'}{a} \mu_k^4\right)$$

Here k is either u or l . The additional restriction is that μ_l must be the solution of the pulse equation less than μ_c and μ_u must be greater than μ_c . If for a given P μ_u comes out greater than one, the expression $\left| \frac{\partial \mu_u^2}{\partial P} \right|$ should be taken as zero.

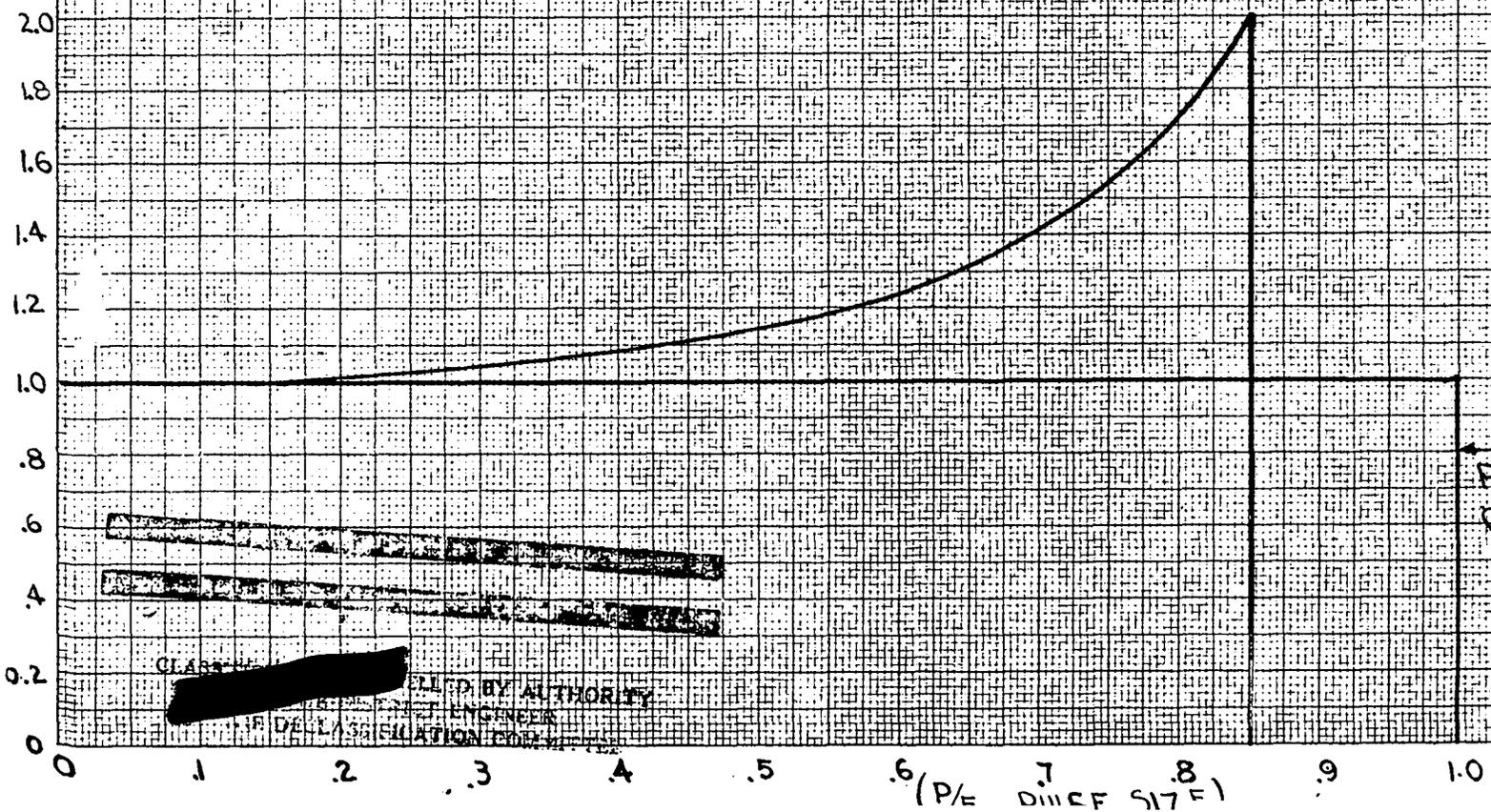
FIGURE 4

DIFFERENTIAL SENSITIVITY CURVE

CONDITIONS:
PLANE PARALLEL COUNTER
THIN PARAFFIN
ELECTRON COLLECTION
INFINITESIMAL CHANNEL

$d =$ CHAMBER DEPTH
 $R_0 =$ RANGE OF PROTONS
OF ENERGY E_0 IN ARGON

$\frac{d}{R_0} = .25$



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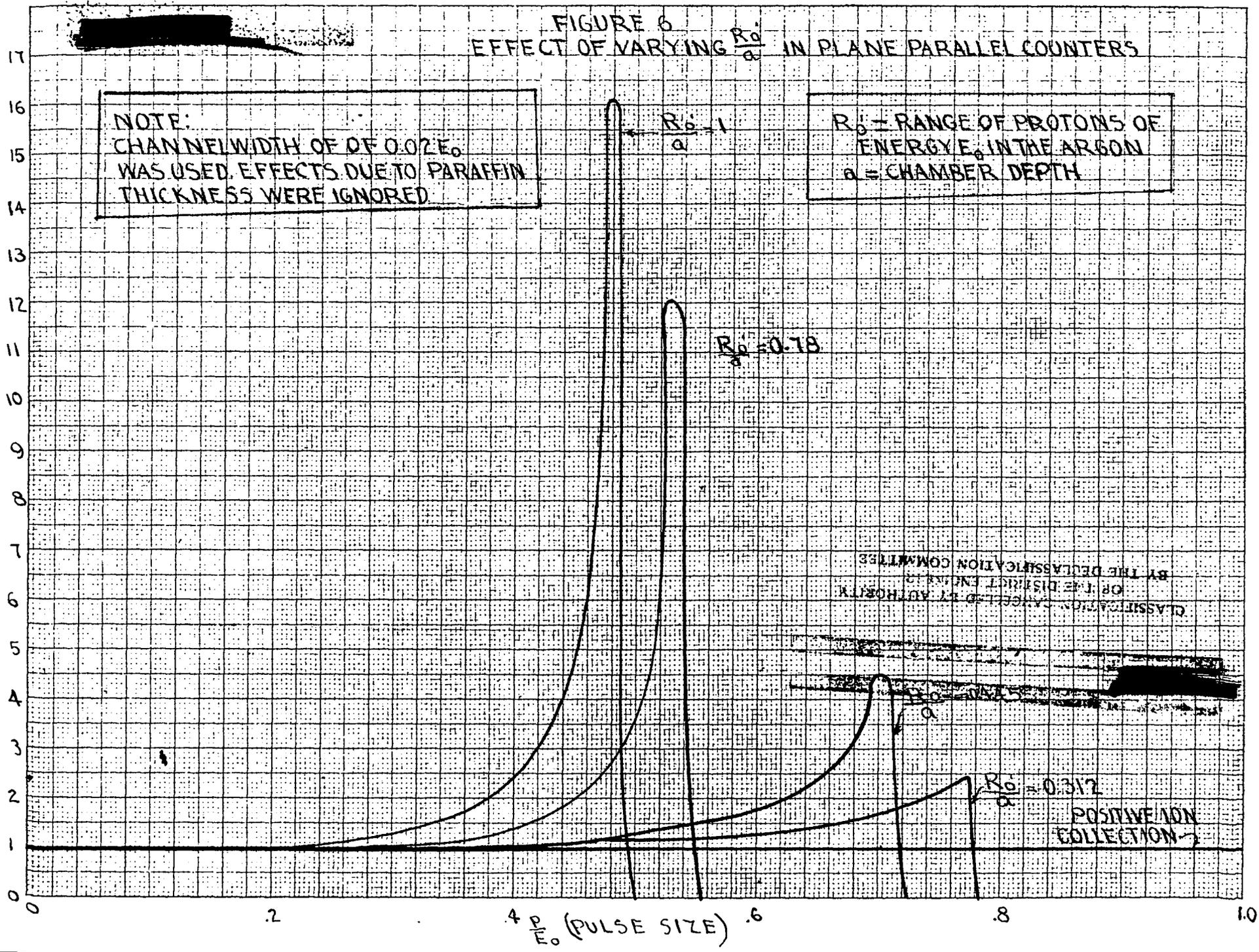
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FIGURE 6
EFFECT OF VARYING $\frac{R_0}{a}$ IN PLANE PARALLEL COUNTERS

NOTE:
CHANNEL WIDTH OF $0.02E_0$
WAS USED. EFFECTS DUE TO PARAFFIN
THICKNESS WERE IGNORED

R_0 = RANGE OF PROTONS OF
ENERGY E_0 IN THE ARGON
 a = CHAMBER DEPTH



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VI. Plane Parallel, Thick Paraffin, Electron Collection

This is the type of plane parallel counter actually used in the laboratory.

As coordinates we take μ and the energy (E_2) with which the proton enters the chamber.

In Section V it was shown that: $P = E_2 \left[1 - \frac{3}{5} \frac{R_0'}{a} \left(\frac{E_2}{E_0} \right)^{3/2} \mu \right]$. The formula $x = a \mu \left[E_0^{3/2} \mu^3 - E_2^{3/2} \right]$ was found in Section IV. Since the number of recoils in the region x , μ is $nv \sigma_H dx d\mu^2$ the number in the region E_2 , μ is equal to $nv \sigma_H \left| \frac{\partial x}{\partial E_2} \right| d\mu^2 dE_2$. Using the relation between x and E_2 , the number in the region E_2 , μ becomes $\frac{3}{2} nv \sigma_H \mu a E_2^{1/2} dE_2 d\mu^2$. Introducing P as a variable instead of E_2 yields:

$$\begin{aligned} \text{D.S.}(P) &= \frac{1}{M} \int_{\mu_1}^{\mu_2} 3 nv \sigma_H a \mu^2 E_2^{1/2} \left| \frac{\partial E_2}{\partial P} \right| d\mu \\ &= \frac{3 R_0}{t E_0^{3/2}} \int_{\mu_1}^{\mu_2} \frac{\mu^2 E_2^{1/2} d\mu}{1 - (3/2)(R_0'/a)\mu(E_2/E_0)^{3/2}} \end{aligned}$$

since $M = nv \sigma_H t$. μ_1 is the value of μ at $x = 0$ that will produce the given pulse and so is given by: $P = E_0 \left(\mu_1^2 - \frac{3}{5} \frac{R_0'}{a} \mu_1^6 \right)$. Similarly μ_2 is the value of μ at $x = t$ that will yield P . To find it let us consider the equations it must satisfy: for any μ : $x = R_0 \mu \left[\mu^3 - (E_2/E_0)^{3/2} \right]$. Therefore μ_2 must satisfy: $t = \frac{R_0 \mu_2}{E_0^{3/2}} (E_0^{3/2} \mu_2^3 - E_2^{3/2})$. Since it must produce the given pulse it must also fulfill:

$$P = E_2 \left[1 - \frac{3}{5} \frac{R_0'}{a} \left(\frac{E_2}{E_0} \right)^{3/2} \mu_2 \right]$$

From these equations it is possible, given P , to find μ_2 . If, however, this yields $\mu_2 > 1$, the value 1 should be used. Unfortunately the expression for the differential sensitivity cannot be integrated directly and so numerical means must be used. The curve for one such counter has been calculated for Rossi. It is shown in Fig. 7. When integrated it yields the total sensitivity curve of Fig. 8. It is to be noted

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that the two effects of thick paraffin and electron collection combine as expected. Thick paraffin cuts down the number of small pulses while electron collection lowers the maximum pulse and causes a peaking of the large pulses.

For this type counter a range energy relation in the paraffin of the type $R = \alpha E^{3/2} + \beta E$ has also been used. The formulas that result after a calculation very similar to the above are:

$$\text{D.S.}(P) = \frac{3}{t} \int_{\mu_1}^{\mu_2} \frac{\mu^2 (\alpha E_2^{3/2} + (2/3)\beta)}{1 - (3/2)(R_0^2/a) \mu_2 (E_2/E_0)^{3/2}} d\mu$$

Where E_2 is given by: $P = E_2 \left[1 - (3/5)(R_0^2/a) \mu_1^6 \right]$ μ_1 is the solution of: $P = E_0 (\mu_1^2 - (3/5)(R_0^2/a) \mu_1^6)$ and μ_2 is the smaller of 1 and the solution of the pair of equations:

$$t = \mu_2 \alpha \left[E_0^{3/2} \mu_2^3 - E_2^{3/2} \right] + \beta \mu_2 \left[E_0 \mu_2^2 - E_2 \right]$$

$$P = E_2 \left[1 - (3/5)(R_0^2/a) \mu_2 (E_2/E_0)^{3/2} \right]$$

Figs. 9 and 10 show the differential and integral sensitivity curves obtained in a particular case for Staub using these formulas. The principal effect of the addition of the βE term is to keep the differential sensitivity finite at P equal to zero.

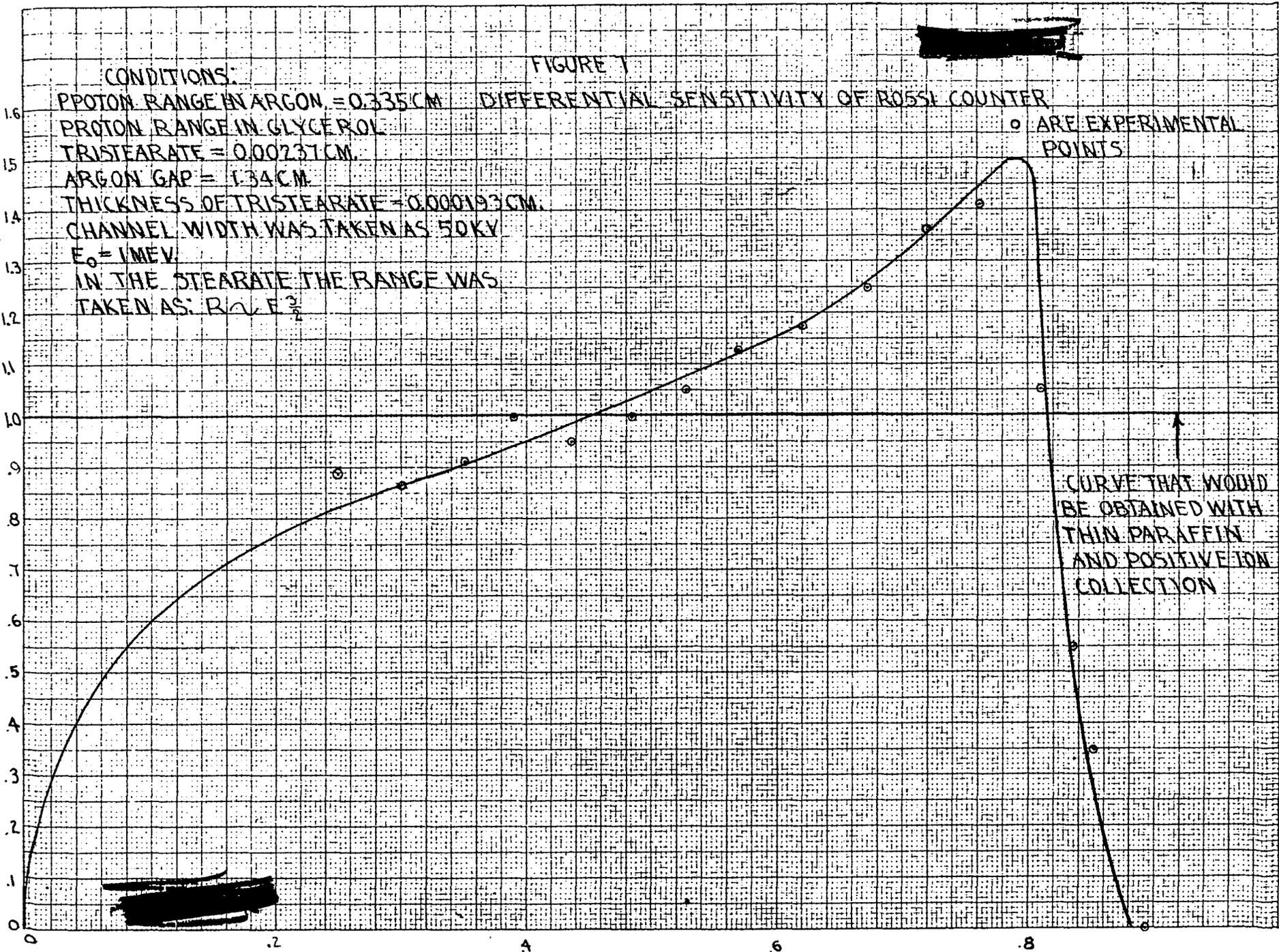
FIGURE 1

DIFFERENTIAL SENSITIVITY OF ROSSI COUNTER

CONDITIONS:

PROTON RANGE IN ARGON = 0.335 CM
PROTON RANGE IN GLYCEROL
TRISTEARATE = 0.00237 CM.
ARGON GAP = 1.34 CM.
THICKNESS OF TRISTEARATE = 0.000193 CM.
CHANNEL WIDTH WAS TAKEN AS 50 KV
 $E_0 = 1 \text{ MEV}$
IN THE STEARATE THE RANGE WAS
TAKEN AS: $R \propto E^{3/4}$

○ ARE EXPERIMENTAL POINTS



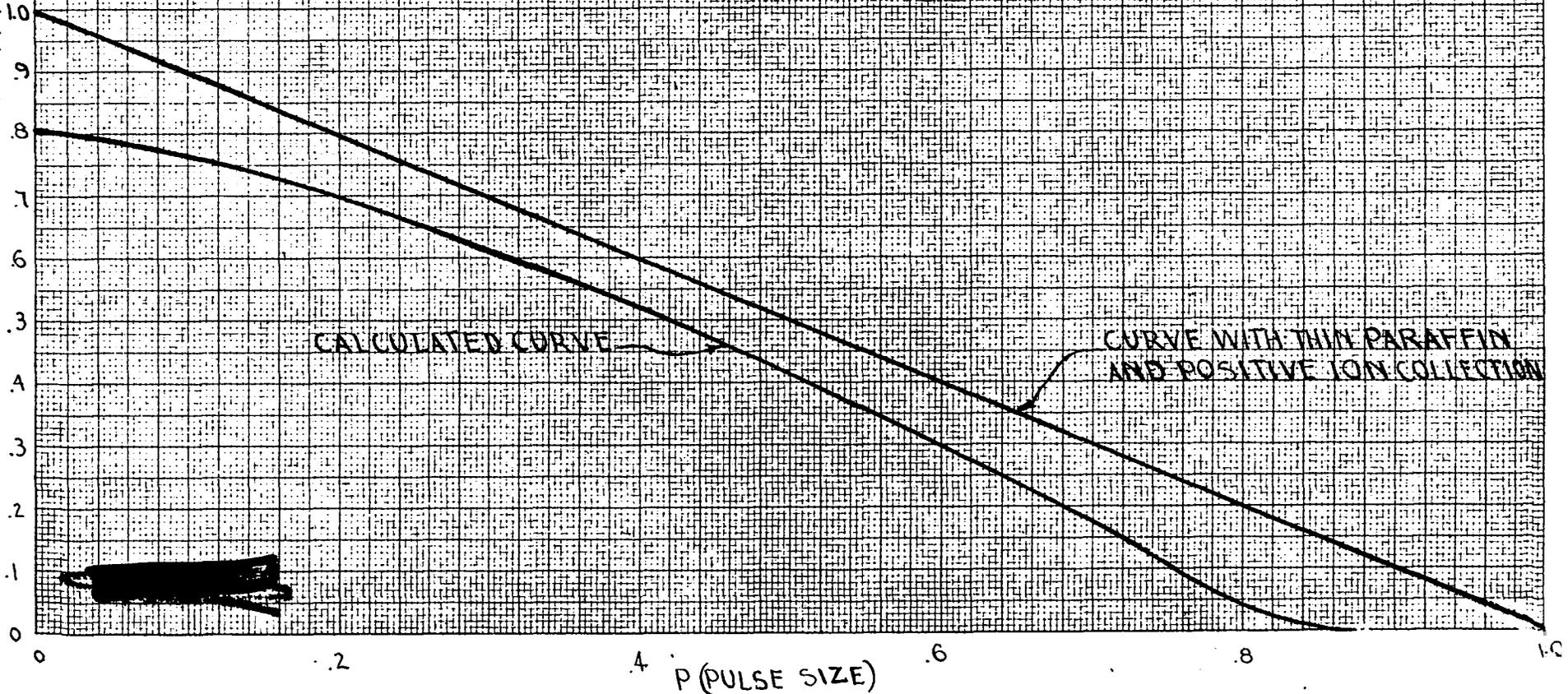
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FIGURE 8
SENSITIVITY VS. PULSE SIZE FOR ROSSI COUNTER

CONDITIONS:
PROTON RANGE IN ARGON = 0.335 CM
PROTON RANGE IN GLYCEROL
TRISTEARATE = 0.00231 CM.
ARGON GAP = 1.34 CM
THICKNESS OF TRISTEARATE = 0.000193 CM
 $E_0 = 1 \text{ MEV.}$
IN THE STEARATE THE RANGE WAS
TAKEN AS: $R \propto E^{\frac{3}{2}}$

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