[redacted] UNCLASSIFIED

LA - 484

This is copy 13 of 15 copies

C.3

December 29, 1945

This document contains 14 pages

THEORY OF ION CHAMBERS

WORK DONE BY:

Thoma M. Snyder

REPORT WRITTEN BY:

Thoma M. Snyder

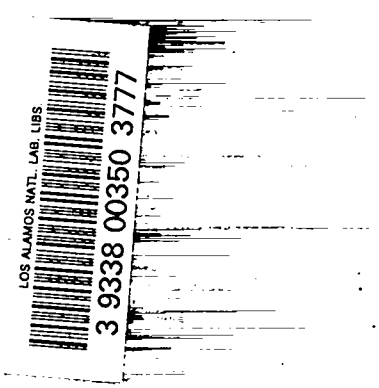
[redacted] UNCLASSIFIED

[redacted]

ABSTRACT

A simple method for calculating the shape of electrical pulses induced in ion chambers by the collection of single ion tracks will be established; then this method will be applied to plane, cylindrical and spherical chamber geometries. From the results obtained, the proper use of these various chambers will become clear and some new methods of application will suggest themselves.

[redacted]



[redacted]

THEORY OF ION CHAMBERS

Consider the general ion chamber as consisting of a high-voltage electrode, a collecting electrode and a guard ring. A fixed potential, V, is applied between the first and last of these electrodes. The collecting electrode is connected to the guard ring through a resistance, R, which is large enough so that its time constant, RC, where C is its capacity, is long compared to the transit time for ions crossing the chamber. The signal obtained on the collecting electrode is applied to the grid of an amplifier tube either directly or through a condenser.

The problem is to calculate the potential change, ΔV, induced in the collecting electrode by a system of charges between the electrodes. This is done by solving Laplace's equation for three auxilliary problems such that their sum is a solution of the problem at hand.

Let A be the potential function for the chamber system when no charges are present between the electrodes and when the floating electrode has its normal potential. Let B be the function describing the system with the electrodes grounded, but after a system of charges has been introduced between them. These charges will induce a charge, $-q$, on the floating electrode. Let C be the solution of the situation when the electrode is floating, both sources of potential are grounded, no charges are between the plates, but a charge, $+q$, is placed on the floating electrode, which puts this at a potential $\Delta V = q/C$. Then A+B+C is the potential function which is the solution to the problem in question; namely, with charges introduced in the chamber and the floating electrode free. A takes care of the charge distributions present at equilibrium, B+C guarantees that the introduction of charges into the chamber has not changed the charge on the floating electrode. A+B+C puts all electrodes of fixed

[redacted]

potential at their proper values. Hence all boundary conditions are satisfied and since A+B+C is a solution of Laplace's equation, it is the only solution. This lead to the theorem: the introduction of a system of charges between the electrodes of an ion chamber induces a change in potential of the floating electrode given by

$$\Delta V = q/C$$

where -q is the image charge that would be induced on this electrode if it were grounded.

This theorem is of especial value when the chamber has plane, cylindrical or spherical symmetry and when there are no edge effects; i.e., when the electrodes are infinite coaxial cylinders or concentric spheres. In this case there is no guard ring. In practice, large planes or long cylinders of small separation will approximate these conditions quite well if none of the ions are near the edges. Under these conditions this theorem is used to reduce the calculation of ΔV to a one-dimensional problem.

Plane Parallel Chamber Neglecting Edge Effects. A) Pulse Shape From A Single Ion Pair

Let the measure of distances from the collecting electrode of a plane parallel chamber be Z. Let d be the plate separation and A the collector area. If a ion pair is introduced at coordinates Z_+, Z_- the potential induced on the collecting electrode is independent of their x,y coordinates unless one approaches the edges so that some charge begins to appear on the guard ring. Hence for convenience we may replace the point charges +e and -e by uniform planes of charge at $Z=Z_+$ and $Z=Z_-$ with charge densities +e/A and -e/A respectively. Now we may compute ΔV as the change in $\int_0^d E_Z\, dZ$. E_Z is changed only between $Z = Z_+$ and $Z = Z_-$ where it is less by the amount $4\pi e/A$. Hence $\Delta V = (4\pi e/A)(Z_- - Z_+)$. But because of the capacity of the collecting

electrode, external to that with respect to the high-voltage electrode, its potential change is reduced by the ratio of the proper chamber capacity to the total capacity so that ΔV is actually given by $\Delta V = e(Z_- - Z_+)/dC$. With this relation the pulse shape caused by collection of a single ion pair is readily obtained. Let the mobilities of the electron and positive ion be K_- and K_+ respectively. Then $dZ_-/dt = -K_- V/d$ and $dZ_+/dt = K_+ V/d$ so that $\Delta V = -(eV/Cd^2)(K_- + K_+)t$ initially. But when one of the particles has been collected (in general, the electron) the behavior is given by

$$\Delta V = -(e/dC)\left\{Z_- + (V/d)\, K_+ t\right\}$$

if the high voltage electrode is negative and

$$\Delta V = (e/dC)\left\{(d - Z_-) + (V/d)\, K_+ t\right\}$$

if it is positive. It is clear that the direction of the field makes no essential difference in the chamber's behavior. The final value of ΔV when both particles are collected is $\Delta V = e/C$ the sign depending on this direction. The whole behavior is given in Fig. 1. The collection time for the electron is $Z_- d/K_- V$ and for the positive ion $Z_+ d/K_+ V$.

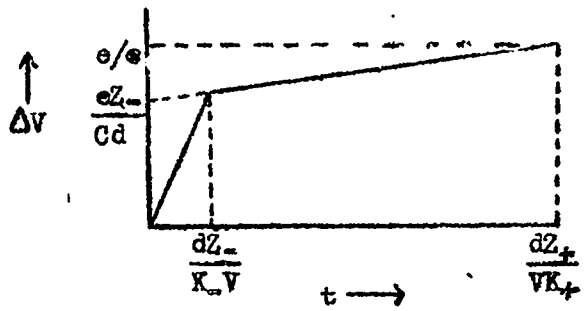


Fig. 1

B) Pulse Shape From An Ion Track

The potential change of the collector caused by an entire ion track is given by adding the effects caused by individual ion pairs. Initially then

$\Delta V = -(neV/Cd^2)(K_+ + K_-)t$. As particles are removed from the motion this expression alters until finally $\Delta V = -ne/C$ so that ΔV gives a measure of the number of ion pairs in the track. Because the track will be linear in the absence of a magnetic field we can give ΔV more precisely. Suppose the track has a range $R \leq d$ and enters the chamber from the high voltage electrode making an angle θ with the normal to the plates. Suppose that the density of ionization along the track is distributed according to some law $\rho = \rho_o\left[(d-Z)/\cos\theta\right]$. We may neglect the effect of the moving positive ions for short times since their velocity is relatively slow, i.e., $K_+ \ll K_-$. Then up to a time $d(d - R\cos\theta)/K_- V = t_1$

$$\Delta V = -(neV/Cd^2)K_- t$$

From then on the number of electrons in motion is given by

$$\int_o^{R-(K_-V/d\cos\theta)(t-t_1)} \rho(\ell)\, d\ell$$

The number collected by time $t \geq t_1$ is

$$\int_{R-(K_-V/d\cos\theta)(t-t_1)}^{R} \rho(\ell)\, d\ell$$

ρ is adjusted so that $\int_o^R \rho d\ell = n$. Therefore

$$\Delta V = -(e/Cd)\int_{R-(K_-V/d\cos\theta)(t-t_1)}^{R} \rho(\ell)\left[d - \ell\cos\theta\right]d\ell$$

$$-(eV/Cd^2)\int_o^{R-(K_-V/d\cos\theta)(t-t_1)} \rho(\ell)\, d\ell\, K_- t$$

When all electrons are collected the value is

$$\Delta V = (e/Cd)\int_o^R \rho(\ell)\left[d - \ell\cos\theta\right]d\ell$$

These expressions do not yield much of value until we differentiate them. Then we obtain at first $\left[d(\Delta V)/dt\right]_{t<t_1} = -(neV/Cd^2)K_-$ so that the initial rise is always pro-

portional to the number of ion pairs present. Later on the shape is given by

$$\left[d(\Delta V)/dt\right]_{t>t_1} = -(eV/Cd^2)K_- \int_0^{R-(K_-V/d\cos\theta)(t-t_1)} \rho(\ell)\, d\ell$$

i.e., is given by the number of electrons still in motion. In particular if the ionization is uniform, i.e., $\rho = n/R$ then

$$d(\Delta V)/dt = -(neV/Cd^2)K_- \left\{1 - (K_-V/dR\cos\theta)(t-t_1)\right\}$$
$$= -(neV/Cd^2)(K_-/R\cos\theta)\left\{d - (K_-V/d)\,t\right\}$$

Still neglecting positive ion motion, we see that the final value of $d(\Delta V)/dt$ is zero. The case of uniform ionization is illustrated in Fig. 2

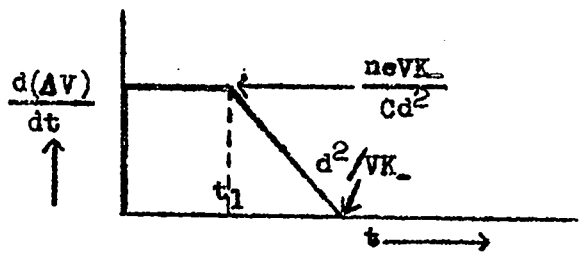


Fig. 2

It is interesting to calculate $d^2(\Delta V)/dt^2$. It is initially very large while the track electrons are getting into a steady drift motion since this is a very short time. During the time t_1, it vanishes. Between the times t_1, and $t = d^2/K_-V$ it has the value

$$\frac{d^2(\Delta V)}{dt^2} = -\frac{eV}{Cd^3}\frac{K_-^2}{\cos\theta}\rho\left[R - \frac{K_-V}{d\cos\theta}(t-t_1)\right]$$

The behavior for ρ constant is shown in Fig. 3.

These considerations have led to suggesting a new way of measuring total ionization in a track; namely by differentiating the pulse. Let us consider this in more detail. If the pulse is first amplified and then differentiated

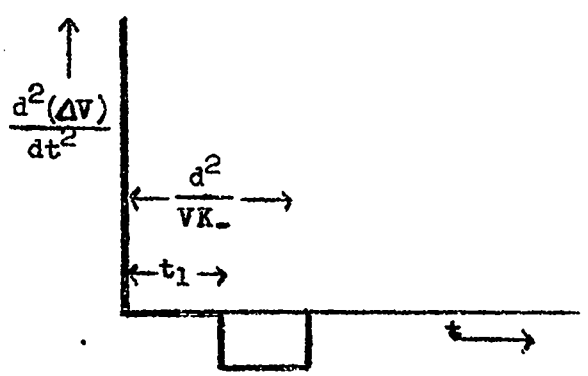


Fig. 3

the resulting pulse will be above the noise level of further amplifying stages that are required. An RC differentiating circuit responds to a voltage V = kt with an output

$$V = kRC\left[1 - e^{-t/RC}\right]$$

Therefore not only must the amplifier be fast compared to the duration of the slope to be measured but also the RC of the differentiating circuit must be small in order for it to respond fully. The easiest thing to do is to use a gas where the electrons are collected somewhat slowly such as pure argon. Here collection times are of the order of a microsecond. By using a deeper chamber this can be increased if all tracks are kept away from the positive electrode. Amplifiers with rise times of .06 μseconds are available. This makes the scheme seem feasible. The loss of pulse height in the differentiating circuit is of the order RC/t where t is the electron collection time. If this is .05 and the original signal to noise ratio obtainable was 100 to 1 (low capacity chamber input and triode first amplifier tube) it will end up as perhaps 10 or 20 to 1, adequate for measurements which do not require more precision than this.

One drawback to this scheme is the more serious edge effects. In the positive ion and electron collection scheme the pulse height caused by a single ion pair is everywhere the same, even at the edges of the collecting electrode.

The question arises as to the effect of the differentiating scheme of collecting ions near the edge of a plate. Consider a rectangular collecting plate. A point charge near the edge may be replaced by a line charge parallel to the edge. We extend the length of plate and line so that the problem becomes two-dimensional.

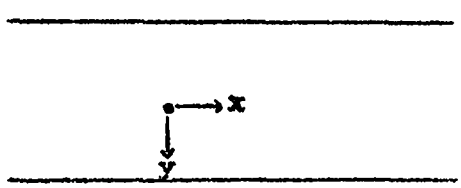


Fig. 4

[redacted]

Now we may use the method of images, replacing the line charge by two grids of parallel line charges. At the collector plate, the x components of field cancel. The y component is of the form

$$E_y = \frac{(2\pi/a)\ \rho \sin (2\pi y/a)}{\cosh (2\pi x/a) + \cos (2\pi y/a)}$$

where a equals 2d and y equals (d-s) for one grid and -(d-s) for the other. The total field is then

$$E_y = \frac{(2\pi/d)\ \rho \sin \left[2\pi(d-s)/2d\right]}{\cosh (\pi x/d) + \cos \pi(d-s)/d}$$

This may also be written as:

$$E_y = -\ 2\rho \sum_{n=1}^{\infty} (4\pi/2d)(-1)^n \sin \left[\pi n(d-s)/d\right]\ e^{-\pi n|x|/d}$$

Now at the surface of the plate, $E_y \neq 0$. The total charge not induced on the collector is:

$$q = \int_{-\infty}^{-x} (E_y/4\pi)\ dx$$

if the edge of the collector has coordinate -x. This is:

$$\rho \sum_{n=1}^{\infty} (1/\pi n)(-1)^n \sin \left[\pi n(d-s)/d\right]\ e^{-\pi n|x|/d}$$

Now

$$(-1)^n \sin \left[\pi n(d-s)/d\right] = (-1)^n \sin (\pi n - \pi ns/d)$$

$$= (-1)^n \left[\sin \pi n \cos \pi ns/d - \cos \pi n \sin \pi ns/d\right] = \sin \pi ns/d$$

Hence this becomes

$$\rho \sum_{n=1}^{\infty} (1/\pi n) \sin (\pi ns/d)\ e^{-\pi ns/d}$$

Now ρ is the charge per unit length of the wires. Our problem is to determine the initial $dv/dt = (1/C)\ dq/dt = (1/C)(dq/dy)\ dy/dt$. Since dy/dt is constant we only have to investigate variations in dq/dy. The following table gives the ratio of dq/dy for ions at points near the edge relative to dq/dy for an ion far from the edge.

[redacted]

distance from collector / distance from edge	0	d/2	d
0	.5	.5	.5
d/2	1.26	.96	.83
d	1.05	1.00	.95

It may be seen that near the edge, the slope dv/dt will be too small when the electron is far from the collector and too large when the electron is too close. Over the bulk of the intermediate region, conditions are much better. Nevertheless, one must exclude a region of width d from the edge, from giving an accurate measurement of the number of ion pairs. For circular plates collector radius R, the ratio of bad to total volumes is 2d/R, calling for a large R/d for accurate work.

It was shown that if second derivatives of the pulses could be taken electronically very sharp spikes would result from each pulse, followed by relatively weak back waves. These pulses would be nearly instantaneous with the formation of the track and give very high pulse resolution and very accurate time location for coincidence work. Actually this does not look as promising because as mentioned before the output of the first differentiating circuit rises with its RC, and is in fact $d(\Delta V)/dt = -(neV/Cd^2)\, K_- RC\left[1 - e^{-t/RC}\right]$. The second derivative in reality becomes

$$d^2(\Delta V)/dt^2 = -(neV/Cd^2)\, K_- \, t\, e^{-t/RC}$$

if the second differentiating circuit has the same RC as the first. This has a maximum value at t equals RC and this value is

$$\left[d^2(\Delta V)/dt^2\right]_{t=RC} = -(neV/Cd^2)\, K_- \, RC/e$$

where $\bar{e}$ = electronic charge, e = transcendental number, and now r = range of particle. Compared to the back kick $eV^2 K_-^2 n(RC)^2/Cd^3 \cos \theta r$. The relative heights are about 20 to 1, for RC equals $20 \times d^2$ KV as chosen. The spike is a factor e closer to noise, the back kick is probably lost in it.

The point is, however, that if a fast collecting gas is used and single differentiation employed just as fast a pulse results.

Cylindrical Ion Chamber

Let the radii of the two coaxial electrodes of a long cylindrical ion chamber be a and b respectively with b>a. Let a potential V be used for ion collection. What is the pulse shape caused by the production of an ion pair at r equals r_o? In this problem we may smear the point charges over a cylinder of radius r equals r_o. One cylinder, usually the negative one, will move towards the inner electrode, the other towards the outer. The change in potential of the collecting electrode is given by integrating the field between the two cylinders of charge and then taking into account the reduction of potential change caused by capacity of the chamber external to itself, just as in the case of the parallel plane chambers. The result is

$$\Delta V = (e/C) \ln(r_+/r_-) \Big/ \ln(b/a)$$

where r_+ and r_- are the radii of the + and - charges respectively. Again when both particles have been collected $\Delta V = e/C$.

Suppose again K_- and K_+ are the electron and positive ion mobilities. Then

$$\frac{dr_+}{dt} = K_+ \frac{V}{\ln(b/a)} \frac{1}{r_+} \qquad \frac{dr_-}{dt} = -K_- \frac{V}{\ln(b/a)} \frac{1}{r_-}$$

and

$$r_+^2 - r_o^2 = 2K_+ \frac{V}{\ln(b/a)} t \qquad r_o^2 - r_-^2 = 2K_- \frac{V}{\ln(b/a)} t$$

This gives

$$\Delta V = \frac{e}{C \ln(b/a)} \ln \sqrt{\frac{r_o^2 + 2K_+ Vt/\ln(b/a)}{r_o^2 - 2K_- Vt/\ln(b/a)}}$$

If the ion pair is not formed too near the central electrode, the initial effect will be largely caused by the electron or approximately

[redacted]

$$\Delta V = \frac{e}{C \ln(b/a)} \ln \frac{r_o}{\sqrt{r_o^2 - 2K_- Vt/\ln(b/a)}}$$

Correspondingly

$$\frac{d(\Delta V)}{dt} = \frac{e}{C \ln(b/a)} K_- \frac{V}{\ln(b/a)} \frac{1}{r_o^2 - 2K_- Vt/\ln(b/a)} = \frac{eK_- V}{C \ln^2(b/a)} \cdot \frac{1}{r_-^2}$$

These relations show no simple behavior except that the effect of an electron ΔV depends on where it is formed and its effect on $d(\Delta V/dt$ depends on where it is. It is clear that most of the effect of a moving charge on ΔV occurs near the inner collector, especially if $b \gg a$. Let us ask whether we can make the inner collector so small that electrons produced in only a relatively small volume of the chamber give rise to a ΔV differing appreciably from that caused by electrons moving all the way across the chamber.

The ΔV arising from a single electron originating at r_o is proportional to $\ln(r_o/a)$. We will ask over what fraction, f, of the volume is $\ln(r_o/a)$ not within 10% of $\ln(b/a)$. This is $f = (r_o^2 - a^2)/b^2$ where $r_o/a = (b/a)^{0.9}$ so if $b = 1024a$, $f = .25$. This is not very promising. However things are somewhat better if we pose our question in a more practical manner. We must use a chamber for this purpose with an amplifier whose long time constant is greater than the time for electron transit from b to a. In this time positive ions formed near the wire, where the electrons alone have too small an effect, can move enough to contribute to the pulse size. The maximum electron collection time is

$$t = \frac{b^2 - a^2}{2K_- V} \ln(b/a)$$

Positive ions move a distance in this time given by $r_+ - r_o$ where

$$r_+ = \left\{(b^2 - a^2)(K_+/K_-) + r_o^2\right\}^{\frac{1}{2}}$$

So that the total potential change in this time is approximately proportional to

$$\ln \left[(b^2 K_+/K_- + r_o^2)^{\frac{1}{2}}/a\right]$$

[redacted]

[redacted]

For small r_o (implying small a) this approaches $\ln\left[(b/a)(K_+/K_-)^{\frac{1}{2}}\right]$. Now $K_- \simeq 100K_+$ so that this limiting value is about .7 $\ln(b/a)$ when b equals 1000a. Actually the circuit responds almost as well for times somewhat longer than t so that conditions are better than this. Furthermore the number of tracks entirely close to the wire is small, in general improving conditions even more. Therefore total ionization in a cylindrical chamber can be measured fairly well in this simple way.

Spherical Ion Chamber

Let a and b be the radii of the two concentric electrodes. This time we smear the charges of a single ion pair over spherical surfaces and handle as before. If the instantaneous radii of positive and negative charges formed at r_o and r_+ and r_- then

$$\Delta V = -\frac{ab}{b-a}\frac{e}{C}\left(1/r_- - 1/r_+\right)$$

$$\frac{dr_+}{dt} = K_+\frac{abV}{b-a}\frac{1}{r_+^2} \qquad \frac{dr_-}{dt} = -K_-\frac{abV}{b-a}\frac{1}{r_-^2}$$

$$r_+^3 = r_o^3 + 3K_+\frac{abV}{b-a}t \qquad r_-^3 = r_o^3 - 3K_-\frac{abV}{b-a}t$$

$$\Delta V = -\frac{abe}{(b-a)C}\left\{\frac{1}{\left[r_o^3 - 3K_-\left(abV/[b-a]\right)t\right]^{1/3}} - \frac{1}{\left[r_o^3 + 3K_+\left(abV/[b-a]\right)t\right]^{1/3}}\right\}$$

and from electrons alone

$$\frac{d(\Delta V)}{dt} = -\left(\frac{a\,b}{b-a}\right)^2\frac{eK_-V}{C}\cdot\frac{1}{r_-^4}$$

It is clear that the most effective region for moving charges is even closer to the central electrode than in the case of the cylindrical chamber. The maximum collection time is approximately (when $b \gg a$)

$$t = b^2(b-a)/3aVK_-$$

In this time positive ions formed near the center collector have drifted to

[redacted]

$$r_+^3 = r_o^3 + (K_+/K_-)\, b^3$$

So that ΔV is given by

$$\Delta V = -\frac{abe}{(b-a)C}\left\{\frac{1}{a} - \frac{1}{\left[r_o^2 + (K_+/K_-)\, b^3\right]^{1/3}}\right\}$$

This has the fractional value of the maximum

$$f \simeq 1 - \frac{\left[1-(K_-/K_+)^{1/3}\right]a}{b-a}$$

If $K_+ = 125K_-$ and b equals 100a this becomes .96 which is quite good. Hence a time constant in the amplifier adequate for electron collection gives a very good proportionality between pulse height and number of ion pairs per track.

UNCLASSIFIED

UNCLASSIFIED