

**STOPPING POWERS FOR USE  
WITH CAVITY CHAMBERS**

Handbook 79



Department of Commerce  
National Bureau of Standards

92  
786  
N6  
1961  
HC

## STOPPING POWERS FOR USE WITH CAVITY CHAMBERS

Stopping Power Study Group

Subcommittee M-3  
Standards and Measurements of  
Absorbed Radiation Dose

Recommendations of the  
National Committee on Radiation Protection  
and Measurements

NCRP Report No. 27



National Bureau of Standards Handbook 79

Issued September 1, 1961

### ARDS

are avail-  
cuments,  
, at the

	Price
Supplement to Screw-Thread Standards for Federal Services 1944.....	\$1.25
28 Screw-Thread Standards for Federal Services 1957, Part I (Amends in part H28 1944 and in part its 1950 Supplement).....	.60
28 Screw-Thread Standards for Federal Services 1957, Part II.....	1.25
30 National Electrical Safety Code.....	.75
37 Testing of Weighing Equipment.....	2.25
42 Safe Handling of Radioactive Isotopes.....	2.50
43 Installation and Maintenance of Electric Supply and Communication Lines. Safety Rules and Discussion.....	.20
44 Specifications, Tolerances, and Regulations for Commercial Weighing and Measuring Devices—2d Edition.....	2.25
45 Testing of Measuring Equipment.....	2.00
48 Control and Removal of Radioactive Contamination in Laboratories.....	1.50
49 Recommendations for Waste Disposal of Phosphorus-32 and Iodine-131 for Medical Uses.....	.15
50 X-ray Protection Design.....	.20
51 Radiological Monitoring Methods and Instruments.....	.20
53 Recommendations for the Disposal of Carbon-14 Wastes.....	.15
54 Protection Against Radiations from Radium, Cobalt-60, and Cesium-137.....	.25
55 Protection Against Betatron-Synchrotron Radiations up to 100 Million Electron Volts.....	.25
57 Photographic Dosimetry of X- and Gamma Rays.....	.15
58 Radioactive-Waste Disposal in the Ocean.....	.20
59 Permissible Dose From External Sources of Ionizing Radiation.....	.35
60 X-ray Protection.....	.20
61 Regulation of Radiation Exposure by Legislative Means.....	.25
62 Report of the International Commission on Radiological Units and Measurements (ICRU) 1956.....	.40
63 Protection Against Neutron Radiation up to 30 Million Electron Volts.....	.40
64 Design of Free-Air Ionization Chambers.....	.20
65 Safe Handling of Bodies Containing Radioactive Isotopes.....	.15
66 Safe Design and Use of Industrial Beta-Ray Sources.....	.20
67 Checking Prepackaged Commodities.....	.35
68 Tabulation of Data on Receiving Tubes.....	1.00
69 Maximum Permissible Body Burdens and Maximum Permissible Concentrations of Radionuclides in Air and in Water for Occupational Exposure.....	.35
72 Measurement of Neutron Flux and Spectra for Physical and Biological Applications.....	.30

## Preface

Values for the stopping power ratio are necessary in order to obtain the energy absorbed in the wall of a cavity chamber from measurements of the ionization produced in the gas of the cavity. The stopping power ratios are also needed in the determination of the exposure dose from ionization measurements in a cavity chamber.

The National Committee on Radiation Protection and Measurement has set up a task group to review the present information on these ratios and to recommend interim values. The task group which prepared this report consisted of the following members:

W. C. Roesch, *Chairman*, General Electric Co., Richland, Wash.  
F. H. Attix, U.S. Naval Research Lab., Washington, D.C.  
M. Berger, National Bureau of Standards, Washington, D.C.  
W. Gross, Columbia University, New York, N.Y.  
G. N. Whyte, Queen's University, Kingston, Ont., Canada

This report has been reviewed for approval by Subcommittee M-3 on "Standards and Measurements of Absorbed Radiation Dose," which consists of the following:

### *Members*

H. O. Wyckoff, *Chairman*  
G. S. Hurst  
H. W. Koch  
H. M. Parker  
W. C. Roesch  
H. H. Rossi  
G. N. Whyte

### *Consultants*

F. H. Attix  
M. Berger  
R. S. Caswell  
D. V. Cormack  
W. Gross  
H. E. Johns  
F. C. Maienschein  
J. W. Motz  
J. A. Sayeg  
R. H. Schuler  
R. W. Wallace

The following parent organizations and individuals comprise the Main Committee:

H. L. Andrews, USPHS and Subcommittee Chairman  
C. M. Barnes, Rep. Amer. Vet. Med. Assoc.  
E. C. Barnes, Am. Ind. Hyg. Assoc.  
C. B. Braestrup, Radiol. Soc. of North America and Subcommittee Chairman  
J. T. Brennan, Col., U.S. Army  
F. R. Bruce, Am. Nuclear Soc.  
J. C. Bugher, Representative at large  
R. H. Chamberlain, Amer. College of Radiology  
W. D. Claus, USAEC  
J. F. Crow, Univ. of Wis.  
R. L. Doan, Am. Nuclear Soc.

C. L. Dunham, USAEC  
 T. P. Eberhard, Am. Radium Soc. and Subcommittee Chairman  
 T. C. Evans, Am. Roentgen Ray Society  
 G. Failla, Representative at large  
 J. W. Healy, Health Physics Soc. and Subcommittee Chairman  
 P. C. Hodges, Am. Medical Assn.  
 E. R. King, Capt., U.S. Navy  
 M. Kleinfeld, Internl. Assoc. Govt. Labor Officials  
 H. W. Koch, Subcommittee Chairman  
 G. V. LeRoy, Subcommittee Chairman  
 D. R. Livermore, Lt. Col., U.S. Air Force  
 W. B. Mann, Subcommittee Chairman  
 W. A. McAdams, Atomic Indust. Forum and Subcommittee Chairman  
 G. W. Morgan, Subcommittee Chairman  
 K. Z. Morgan, Health Physics Soc. and Subcommittee Chairman  
 H. J. Muller, Genetics Soc. of America  
 R. J. Nelsen, Am. Dental Assoc.  
 R. R. Newell, Am. Roentgen Ray Society  
 W. D. Norwood, M. D. Indust. Medical Assoc.  
 J. P. O'Neill, Internl. Assoc. of Govt. Labor Officials  
 H. M. Parker, General Electric Co.  
 C. Powell, USPHS  
 E. H. Quimby, Am. Radium Society and Subcommittee Chairman.  
 J. A. Reynolds, Natl. Electrical Mfgr. Assoc.  
 H. H. Rossi, Subcommittee Chairman  
 M. D. Schulz, Am. College of Radiology  
 T. L. Shipman, Rep. Indust. Med. Assoc.  
 L. S. Skaggs, Subcommittee Chairman  
 Curt Stern, Genetics Soc. of America  
 J. H. Sterner, Am. Indust. Hygiene Assoc.  
 R. S. Stone, Radiol. Soc. of North America  
 L. S. Taylor, NBS  
 E. D. Trout, Natl. Electrical Mfgr. Assoc.  
 B. F. Trum, Rep. Am. Vet. Med. Assoc.  
 Shields Warren, Representative at large  
 J. L. Weatherwax, Representative at large  
 E. G. Williams, Representative at large  
 H. O. Wyckoff, Subcommittee Chairman

The following are the NCRP Subcommittees and their Chairmen:

- Subcommittee 1. Permissible Dose from External Sources, H. M. Parker.
- Subcommittee 2. Permissible Internal Dose, K. Z. Morgan.
- Subcommittee 3. X-rays up to Two Million Volts, T. P. Eberhard.
- Subcommittee 4. Heavy Particles (Neutrons, Protons, and Heavier), H. H. Rossi.
- Subcommittee 5. Electrons, Gamma Rays and X-rays Above Two Million Volts, H. W. Koch.
- Subcommittee 6. Handling of Radioactive Isotopes and Fission Products, J. W. Healy.
- Subcommittee 7. Monitoring Methods and Instruments, H. L. Andrews.
- Subcommittee 8. Waste Disposal and Decontamination. (This subcommittee has been inactivated.)
- Subcommittee 9. Protection Against Radiations from Ra, Co<sup>60</sup>, and Cs<sup>137</sup> Encapsulated Sources, C. B. Braestrup.

- Subcommittee 10. Regulation of Radiation Exposure Dose, W. A. McAdams.
- Subcommittee 11. Incineration of Radioactive Waste, G. W. Morgan.
- Subcommittee 12. Electron Protection, L. S. Skaggs.
- Subcommittee 13. Safe Handling of Bodies Containing Radioactive Isotopes, E. H. Quimby.
- Subcommittee 14. Permissible Exposure Doses under Emergency Conditions, G. V. LeRoy.
- Subcommittee M-1. Standards and Measurement of Radioactivity for Radiological Use, W. B. Mann.
- Subcommittee M-2. Standards and Measurement of Radiological Exposure Dose, H. O. Wyckoff.
- Subcommittee M-3. Standards and Measurement of Absorbed Radiation Dose, H. O. Wyckoff.
- Subcommittee M-4. Relative Biological Effectiveness, V. P. Bond.

A. V. ASTIN, *Director.*

# Stopping Powers for Use with Cavity Chambers

## 1. Introduction

The purpose of this report is to present a critical review of the literature concerning the stopping power ratio that is used in the interpretation of cavity ionization measurements in radiation dosimetry.

The Bragg-Gray principle, which is discussed in detail, is the basic formula relating the ionization in a cavity chamber to the energy absorbed in the chamber wall material. One of the terms in this formula represents the ratio of energy absorption in the wall material to that in the gas. This term is called the stopping power ratio and will be denoted by  $s$  or  ${}_m s$ , when speaking of energy absorbed per cubic centimeter or per gram, respectively. It is the evaluation of this term with which we are primarily concerned here.

It is shown in the development of the Bragg-Gray principle that the interpretation of the  $s$  term has gradually changed over the years. Gray's treatment considered the term as simply the ratio of continuous electron stopping powers in wall material to gas, ignoring the energy dependence of the stopping power. This may be regarded as a first approximation to  $s$ . Laurence later took into account the energy dependence of the stopping power, thus taking for  $s$  a mean value of the stopping power ratio evaluated for the spectrum of electrons crossing the cavity. This constitutes a second approximation to  $s$ , more accurate than the first, but still ignoring the production of fast "secondary" electrons ( $\delta$  rays) by electron-electron collisions. Finally the treatments of Burch and of Spencer and Attix take the latter effect into account to give a third approximation to  $s$ .<sup>1</sup>

It should be emphasized that, where the gas and cavity wall are fairly close in atomic number, the errors incurred by the use of the second approximation in place of the third are small. Even with as great a mismatch as air and aluminum the differences from  $s$  as predicted by Spencer and Attix are only about 1 percent for cavities 0.6 cm in diameter, or 4 percent for cavities 0.1 cm in diameter (at one atmosphere pressure). With graphite or "air-equivalent" walls they predict a difference of the order of one-tenth of the above figures.

In principle, considerations similar to those that follow apply to any ionizing radiation. However the emphasis in

<sup>1</sup>The next approximation, as yet nonexistent, would be one in which the modifying effect of the cavity gas upon the crossing electron flux is also taken into account. This would allow the application of cavity theory to larger cavities than is now possible.

the discussion has been placed upon X- and gamma radiations with only occasional reference to beta radiation, electron beam radiation, and neutrons.

The development of cavity theory is described, more or less chronologically. Then a review of the theory and experimental information on ranges and stopping powers of charged particles is made to provide the data necessary for applying the theories. The information available from cavity chamber measurements is reviewed and compared with theory. Finally, conclusions as to what this study group considers to be the best currently available information about  $s$  are made.

## 2. Cavity Chamber Theory

### 2.1. Stopping Power

If the energy of a charged particle changes an average of  $dT$  in a distance  $dx$  along its path, then the (linear) stopping power,  $S$ , is defined as  $S = -dT/dx$ . The mass stopping power,  ${}_mS$ , is defined by  ${}_mS = S/\rho$  where  $\rho$  is the density. In some studies it is desirable to exclude from the average energy loss the energy lost in discrete amounts greater than some value  $\Delta$ . This exclusion will be denoted by including  $\Delta$  among the arguments of the quantity. For example,  $S(T, \Delta)$  is the stopping power excluding the energy lost in discrete amounts greater than  $\Delta$ .

The unit of energy will be the electron volt (ev) or the meg-electron volt (Mev).  $1 \text{ ev} = 1.602 \times 10^{-12} \text{ erg}$ . The other units will be those of the familiar CGS system.

### 2.2. Absorbed Dose

A fundamental problem in radiation dosimetry is the measurement of the energy imparted to matter by radiations such as X- or gamma rays, fast neutrons, or beta rays.

It will be worthwhile elaborating upon the phrase "energy imparted to matter," for the sake of clarity. First of all, this is meant in a macroscopic sense, referring to regions large enough to represent the average energy transfer of many individual events. At the same time one speaks of the energy absorbed "at a point," by which one means the average value per unit mass in a small region surrounding the point. The accepted unit of this absorbed energy per unit mass, or "absorbed dose" is the rad (100 erg/g) (ICRU, 1956).

The actual mechanisms for the transfer of energy from the radiation to the target material are important in interpreting the meaning of "energy imparted to matter." With beta rays (i.e., fast electrons) the transfer is direct, taking the form of a series of Coulomb-force interactions between the passing electron and the atomic electrons in the irradiated material. In the wake of the fast electron is left a string of atoms that have either been ionized or left in an excited state. Roughly half of the energy is invested in ionization and half in excitation. When (and if) the excited and ionized atoms return to normal energy states, the energy they lose is finally degraded to thermal motion<sup>2</sup> that is measurable at least in principle by calorimetric methods.

The above description applies equally well to the energy spent by X-rays in traversing material *after* the initial transfer of energy from electromagnetic quantum form to electron kinetic energy. Similarly a fast neutron transfers its energy to a nucleus (usually hydrogen, if present) by collision, whereupon the nucleus, stripped of some of its electrons, passes through the material ionizing and exciting atoms by Coulomb interactions as the electron did in the previous cases.

It is important to observe that in the case of X-rays or neutrons, when one refers to the "energy imparted to matter" or "absorbed dose," it is the deposition of energy by the ionizing particle that is meant rather than the *initial transfer* of energy to that particle. Except in the special case of secondary particle equilibrium, the two will differ in magnitude at a given location.

### 2.3. Ionizing Particle Equilibrium

In the case of X- and gamma rays "ionizing particle equilibrium" is usually referred to as "electronic equilibrium" since the ionizing particles produced in that case are electrons. For neutrons the corresponding particles are predominantly protons in most instances. We will discuss the case of X- and gamma rays for convenience of terminology.

There are two types of electronic equilibrium, "complete" and "transient." The former is said to exist at a point when, for every electron leaving a volume element surrounding the point, another electron of the same energy enters. (Strictly, this can be true only in the sense of a statistical limit.) This condition is found only in the case of an extended uniform medium in which a radioactive emitter is

<sup>2</sup> Note that if the ions are not allowed to recombine, if some of the energy is spent in changing the chemical or physical structure or in triggering exothermic or endothermic reactions, or if some of the energy escapes by radiation then the thermal heating will not be a true indication of the energy imparted by the radiation.

uniformly distributed. Complete electronic equilibrium will then exist at internal points sufficiently far from the boundary so as to be unaffected by it. Complete electronic equilibrium implies that the energy lost by electrons within a volume element is equal to the kinetic energy (initial energy imparted minus that lost in coming out of the atom) of the electrons released within the volume element.

When a beam of X- or gamma rays or neutrons enters a medium, the energy lost by the electrons or protons per unit volume at a location near the surface is less than the kinetic energy released per unit volume at the same location. The proportion between the two increases with depth until "transient" electronic equilibrium exists; i.e., the ratio of the energy absorbed to that released within an elementary volume reaches a *constant value* independent of position along the beam direction. Brysk (1954) and later Roesch (1958) showed that a constant ratio does occur. Furthermore the ratio is greater than 1; i.e., the *absorbed* energy at the point is the larger. In many cases this ratio is very close to 1.00, and complete electronic equilibrium is then said to exist, although it is in fact only approximated. Such situations are frequently encountered for X- and gamma rays below a few Mev and for neutrons below several tens of Mev.

#### 2.4. The Fano Theorem

Before embarking upon a chronological review of the principal developments in cavity theory, it will be worthwhile to present a fundamental theorem upon which the Bragg-Gray relation often depends.

Fano (1954) pointed out that in many cases cavity chambers cannot be made small compared to the secondary electron ranges as is conventionally required for application of cavity theory. He stated that the application of the Bragg-Gray principle actually rests, more frequently, on another basis: the flow of corpuscular radiation will remain undisturbed by the presence of the cavity, provided that the elemental compositions of the gas and the surrounding material are identical, regardless of cavity size. Fano proved this in general, with mathematical rigor.

Fano's theorem is stated as follows: "In a medium of given composition exposed to a uniform flux of primary radiations (such as X- or gamma rays or neutrons), the flux of secondary radiation is also uniform and independent of the density of the medium as well as of the density variations from point to point."

Fano's mathematical treatment consists essentially of an application of the following reasoning to each infinitesimal volume of the medium. Consider first an infinite medium of uniform density with a radiation source also distributed with uniform density. The emission of the source need not be isotropic. Throughout the medium there will be a uniform flux of all the secondaries of the radiation. Suppose now that the density of the medium is increased  $f$ -fold so that the source intensity is increased by a factor  $f$ . Suppose that at the same time all the stopping powers become  $f$  times larger; i.e., that the mass stopping powers are independent of density. The resulting flux of any secondary remains unchanged. Finally consider the medium as subdivided into two (or more) parts of different density with corresponding source strengths. Each portion "knows" about the others only through the flux at the boundary. Since this flux is independent of the density, the flux in any portion of the material remains fully unaffected by changes in density in other portions.

There is a limitation on the applicability of Fano's theorem. An  $f$ -fold change in density that increases the source density by  $f$  may *not* increase the stopping powers by  $f$ . The polarization effect results in reduced rates of energy loss in condensed media (see sections 3.1 and 3.3.b and c). Consider again the two (or more) part system. On one side the density is  $f$  times that on the other so that the source intensity is increased  $f$ -fold. In the presence of the polarization effect the stopping powers of the secondaries are not, however,  $f$  times larger in the denser material. This is because the stopping power per electron in the denser material is less than that in the other material. Hence to generate the same flux of secondaries on both sides of the boundary, the elemental composition of the denser material must be adjusted to make the stopping powers exactly  $f$  times smaller while maintaining the  $f$ -fold increase in source intensity. A third region of still another density would have to have yet a different composition, and so on. The situation is even further complicated by the fact that the ratio of the stopping powers of different materials is a function of the electron energy. Thus the flux of only one energy group of secondary electrons could be made equal on both sides of the boundary with a given selection of compositions.

In a frequently encountered situation, a solid-wall cavity chamber irradiated by gamma rays in the energy range where Compton effect predominates, the source strength of secondary electrons is everywhere proportional to the electron density regardless of the atomic number. Thus, in general,

it will not be possible to select a wall material of the right composition simultaneously to increase the source strength by a factor  $f$  while decreasing the electron range by the reciprocal of that same factor.

One must conclude then, that the Fano theorem holds only to the extent that polarization effect does not enter the picture. This effect can be large at high energies in solid materials. In carbon the electron stopping power is altered by 3 percent at 1 Mev and 12 percent at 10 Mev. However at large energies where the polarization effect is large, the electron ranges are large also, and it is easier in practice to construct cavity chambers small in comparison. Thus the Fano theorem usually need not be relied upon in cases where it is least applicable.

### 2.5. Basic Bragg-Gray Principle

The cavity ionization chamber offers a method for obtaining the absorbed dose at a point by a computation based upon a measurement of the ionization produced at the point. In order to allow collection of this ionization by an electrostatic field, the cavity contains a gas, which in general differs in stopping power from the (solid or liquid) wall material. The effect of the stopping power difference on the ionization must be taken into account in the computation. A suitable formula was first devised in essence by Bragg (1910), and later independently devised in more explicit form by Gray (1929, 1936). This Bragg-Gray formula will first be discussed in its basic form, and then the later elaborations of Laurence, of Spencer and Attix, and of Burch will be considered.

Cavity theory may also be applied to detectors such as scintillators, chemical dosimeters, etc., in which the energy imparted to the cavity material is deduced from measurements of light emission, extent of chemical reaction, etc. When the cavity is filled with a solid or liquid, its dimensions must be about three orders of magnitude smaller than a similar gas cavity.

#### a. Gray's Derivation

Gray's treatment of the cavity theory was based upon the assumption that an electron traversing a solid medium loses the same amount of energy in a distance  $\Delta X$ , short compared with its range, as it would lose in traversing a distance  $s\Delta X$  of air, where  $s$  is a proportionality factor that is independent of the velocity of the particle. He then considers two small

geometrically similar volumes of arbitrary shape. Volume  $V_z$  contains the solid material  $Z$ , while  $V_a$  contains air and has all its linear dimensions greater than those in  $V_z$  by the constant factor  $s$ . If these two volumes are each embedded in a large region containing the solid  $Z$ , and exposed to identical, uniform fields of gamma rays, each will be traversed by a flux of electrons from the surrounding material  $Z$ . On account of the larger surface area, the number of electrons entering  $V_a$  will be greater by the factor  $s^2$ , but the volume of  $V_a$  exceeds  $V_z$  by the factor  $s^3$ . Hence the energy spent in the air per cubic centimeter will be less than that in the solid volume by the factor  $s^{-1}$ . Thus we can write

$$E_a = \frac{1}{s} E_z, \quad (1)$$

where  $E_a$  and  $E_z$  are the energies lost by the electrons per cubic centimeter in traversing the air and solid volumes, respectively. (At this point in Gray's argument his terminology has changed from "energy lost" by the electrons to "energy absorbed" in the media. This switch is only valid if the two terms are equivalent. More will be said of this later, but meanwhile we also assume this equivalence.)

$E_a$  can be further expressed as  $wJ$  where  $J$  is the number of ion pairs formed per unit volume of air, and  $w$  is the average energy expended in the air by the electrons per ion pair produced. This yields the familiar Bragg-Gray relation:

$$E_z = swJ. \quad (2)$$

We will not concern ourselves in detail with  $w$ . Gray was led, by the experimental evidence available to him, to the conclusion that  $w$  had a constant value of 32.5 ev, irrespective of the electron velocity. Present evidence would alter the value to a little less than 34 ev, but the constancy still appears to be valid, at least for electrons with initial energies above 20 kev. In any case the essence of the Bragg-Gray principle is contained in equation (1), which relates the energy lost by electrons in the air to that in the solid. The further expression of  $E_a$  in terms of ionization is a secondary step that need not be discussed further here.

#### b. The Nature of $s$ in the Gray Derivation

Gray initially calls  $s$  merely "a proportionality factor which is independent of the velocity of the particle." He further identifies it, however, as the ratio of the stopping

power in the solid material to that in air,  $s = (dT/dx)_z \div (dT/dx)_a$ , by the statement that "a beta particle traversing a solid medium loses the same amount of energy in a distance  $\Delta X$  as it would do in traversing  $s\Delta X$  of air," where  $\Delta X \rightarrow dx$  for infinitesimal cavities, as he later specifies.

The derivation does not explicitly require  $dT/dx$  to represent a *continuous* (frictionlike) energy loss. However, if one allows  $dT/dx$  to include large discrete energy transfers, the resulting energetic secondary electrons can carry some energy out of  $V_a$  or  $V_z$  after such energy has been counted as having been "lost" within those volumes. Thus the energy actually absorbed within  $V_a$  and  $V_z$  would be less than that lost there by the traversing electrons. Under these conditions there would be no *a priori* guarantee that the absorbed energies, say  $E'_a$  and  $E'_z$ , would be related by equation (1), even though  $E_a$  and  $E_z$  might be. One would have first to prove that the energies  $(E_a - E'_a)$  and  $(E_z - E'_z)$  carried out of  $V_a$  and  $V_z$  by secondaries originating within were also related by equation (1). This could be done by showing that, in similar electron paths across  $V_a$  and  $V_z$ , the production of secondaries is alike in corresponding path elements  $s dx$  and  $dx$ , and also that the stopping powers experienced by those secondaries before escaping from  $V_a$  or  $V_z$  are again related by the factor  $s$ .

Consider the first of these two requirements, and for the sake of argument assume the solid to be of higher atomic number than air. Spencer and Fano (1954) proposed that the production of secondaries is given approximately by the Møller formula for knock-on collisions (Møller, 1931), and that it is thus dependent only on the number of atomic electrons present per cubic centimeter, irrespective of  $Z$ . Consequently more secondaries will be produced in the path element  $dx$  in the solid than in element  $s dx$  in the air, because more atomic electrons will be encountered in traversing the elemental distance in the solid. This results from the fact that the electronic stopping power in the higher-atomic-number solid is less than that in air.<sup>3, 4</sup> If  $N_a$  and  $N_z$  are the number of electrons per cubic centimeter in the air and solid, respectively, then

$$\left(\frac{dT}{N_s dx}\right)_z < \left(\frac{dT}{N_a dx}\right)_a,$$

or

$$s = \frac{(dT/dx)_z}{(dT/dx)_a} < \frac{N_z}{N_a}. \quad (3)$$

<sup>3</sup> See Sec. 3 for more details about stopping powers.  
<sup>4</sup> For the moment we neglect the polarization effect.

$N_z dx$  electrons per square centimeter will be encountered in traversing  $dx$ , while  $N_a s dx$  electrons per square centimeter will be encountered in  $s dx$ . The ratio of secondary electron production in the two path elements is  $N_z/sN_a$ , but from eq (3) above, this is greater than unity.

Turning now to the second point, we examine whether the stopping powers experienced by those secondaries before escaping from  $V_a$  or  $V_z$  are related by the factor  $s$ . The probability of production of a secondary electron increases rapidly the lower the energy of the secondary. Thus they will generally be much lower in energy than the primary electron that produced them. Since the linear dimensions of  $V_a$  and  $V_z$  are adjusted to be in the ratio  $s$ , the stopping power ratio for the traversing electrons, one might question whether the same  $s$  still applies to the secondaries. In Gray's derivation this point is inherently covered by the assumption that  $s$  must be independent of the energy. However that assumption is untenable except as a rough approximation in the light of the experimental evidence on stopping power.<sup>5</sup>  $s$  is actually found to decrease as the electron energy decreases, if the solid is greater in atomic number than the air. Thus, of two identical secondaries generated in corresponding electron path elements  $dx$  and  $s dx$  in  $V_z$  and  $V_a$ , respectively, the latter secondary will lose the more energy before leaving its volume and will consequently carry less energy out of the volume. The ratio  $(dT/dx)_z \div (dT/dx)_a$  for the typical secondary is simply less than the  $s$  that relates the linear dimensions of  $V_z$  and  $V_a$ .

On the basis of the above two arguments it can be seen that if  $Z_z > Z_a$ : (a) more secondary electrons are produced by an electron crossing  $V_z$ , and (b) each secondary produced in  $V_z$  carries a larger fraction of its energy out of the volume than does a corresponding secondary in  $V_a$ . Thus, due to the combined effect of (a) and (b), each electron traversing  $V_z$  will have a larger fraction of its lost energy removed from that volume by secondaries than will an electron traversing  $V_a$  in a corresponding path. These two fractions would have to be the same in order that  $(E_z - E'_z) = s(E_a - E'_a)$ .

Thus it has been shown that Gray's derivation of equation (1) as a relation of energies *absorbed* is not valid unless one requires that  $(dT/dx)$  include only continuous energy losses.

One might be tempted to argue that equation (1) could be corrected by adjusting  $s$  to some average value that would take into account the secondaries as well; e.g., in the above

<sup>5</sup> Gray evidently included the assumption for ease of discussion in relation to his dual-volume model. It was thus possible for him to avoid dealing explicitly with the spectrum of the traversing electrons. It was Laurence (1937) who first derived the Bragg-Gray relation without the restriction that  $s$  be a constant.

case where  $Z_2 > Z_1$ , to choose a somewhat larger value for  $s$ . Such an approach has been pursued by Burch (1955) and by Spencer and Attix (1955). It leads to the inevitable conclusion that the volumes  $V_a$  and  $V_z$  cannot be dismissed as merely "infinitesimal," but must be specified in relation to the ranges of the secondary electrons originating within them. Thus if secondaries are allowed to enter the picture,  $s$  unavoidably becomes a function of the cavity size. This consequence will be discussed more fully later.

## 2.6. Later Developments

### a. Laurence's Derivation

As mentioned in footnote 5, Laurence (1937) did not require  $s$  to be a constant with respect to electron velocity as did Gray. Neither did he employ the same model as Gray. Instead he compared the ionization produced in two identical small air-filled cavities, one having solid walls of material  $Z$ , the other gaseous air walls. Both are embedded in large regions of solid or of air and irradiated by identical uniform fields of gamma rays. Even in the absence of photoelectric effect the electron fluxes traversing the two cavities are not the same, because the ranges of the electrons (in electrons/cm<sup>2</sup>) will in general differ because of the different atomic numbers of the surrounding materials. Laurence takes this into account by considering the flux coming from all the production sites out to the maximum electron range from each cavity and allowing for the energy lost by each electron before arriving at the cavity. He first expresses the total ionization in terms of a function  $F(r_w - x)$ , which is the number of ion pairs produced per centimeter of path in a cavity for an electron that originated at a distance  $x$  from the cavity with an initial range  $r_w$  in the wall material ( $Z$  or air). This function  $F$  is then replaced by the substitution  $F(r_w - x) dx = G(T) (dT/dx)^{-1} dT$  where  $G(T)$  is the number of ion pairs per centimeter produced in a cavity by an electron that enters the cavity with energy  $T$ . Thus the total ionization

$$J \propto \int_0^{T_{max}} \left( \frac{d\mu}{dT_0} \right)_w \int_{r_w}^0 F(r_w - x) dx dT_0 \\ = \int_0^{T_{max}} \left( \frac{d\mu}{dT_0} \right)_w \int_0^{T_0} \frac{G(T)}{(dT/dx)} dT dT_0 \quad (4)$$

where  $(d\mu/dT_0)_w$  is the rate of production in the wall material per cubic centimeter, per gamma ray per square centimeter, of electrons having initial energies between  $T_0$  and  $T_0 + dT_0$ . Notice that in making the change of variables the term  $(dT/dx)$  has appeared in the denominator. This is the stopping power for electrons of energy  $T$  in the wall material. Spencer and Fano (1954) and others have pointed out that  $(dT/dx)^{-1}$  closely approximates the spectral distribution of the electron flux at a point about which there is electronic equilibrium, provided that the energy losses are required to be continuous (or infinitesimal). Thus in Laurence's derivation the ionization in either cavity depends on the product of the specific ionization  $G$  (proportional to the stopping power of the air) in the cavity and the traversing electron flux from the wall. In both of these quantities the production of delta rays has been ignored. This point will come up again and be discussed further in the Spencer-Attix derivation.

Laurence expresses his "correction factor for wall effects," as

$$B = \frac{\int_0^{T_{max}} \left( \frac{d\mu}{dT_0} \right)_z \int_0^{T_0} \left[ G_f \left( \frac{dT}{dx} \right)_z \right] dT dT_0}{\int_0^{T_{max}} \left( \frac{d\mu}{dT_0} \right)_a \int_0^{T_0} \left[ G_f \left( \frac{dT}{dx} \right)_a \right] dT dT_0}, \quad (5)$$

which is the ratio of ionization/cubic centimeter in the air cavity with walls of material  $Z$  to that in the identical air-walled cavity.

Equation (1) can be shown to be a special case of equation (5) in the following way. First we make the assumption, as did Gray, that  $w$  and the ratios of stopping powers are both independent of electron energy. Thus  $G/(dT/dx)$  is a constant which can be removed from the integrals, and the  $G$ 's canceled. The energy absorbed per cubic centimeter in a material under equilibrium conditions, when exposed to one quantum  $h\nu$  of gamma rays per square centimeter, is

$$\int_0^{T_{max}} \left( \frac{d\mu}{dT_0} \right) T_0 dT_0 = \mu \bar{T}_0 = \mu_{en} h\nu, \quad (6)$$

where  $\mu_{en}$  is the energy absorption coefficient (in cm<sup>-1</sup>) and  $\bar{T}_0$  is the average energy given to an electron in an interaction. Thus equation (5) can be reduced to

$$B = \frac{\left( \frac{dT}{dx} \right)_a (\mu_{en})_z}{\left( \frac{dT}{dx} \right)_z (\mu_{en})_a} = \frac{1}{s} \frac{(\mu_{en})_z}{(\mu_{en})_a}, \quad (7)$$

where Gray's definition of  $s$  is made use of in obtaining the last equality.

The connection between the dual-volume model used by Gray and that used by Laurence can now be recognized by imagining both models to be exposed to the same field of gamma rays. The absorbed energy<sup>6</sup> per cubic centimeter,  $E_a$ , in the two air cavities with solid walls would be identical. The absorbed energy per cubic centimeter ( $E_{aa}$ ) in the uniform air (Laurence's air-filled "cavity" with air walls) would be  $(\mu_{en})_a/(\mu_{en})_z$  times that ( $E_z$ ) in the uniform solid (Gray's solid-filled "cavity" in solid walls). Thus equation (7) becomes

$$B = \frac{E_a}{E_{aa}} = \frac{E_a}{(\mu_{en})_a E_z} = \frac{1}{s} \frac{(\mu_{en})_z}{(\mu_{en})_a} \quad (8)$$

which reduces to equation (1). Gray's equation is thus shown to be a special case of Laurence's where  $w$  and  $s$  are taken to be independent of electron energy.

#### b. Derivation of Basic Bragg-Gray Principle by Spencer and Attix

Spencer and Attix (1955) considered a single small air cavity within an extended solid medium under uniform gamma radiation. Electrons of initial energy  $T_0$  Mev are assumed to be generated at a rate 1 per g per sec everywhere in the solid. Thus, as electronic equilibrium exists inside the solid, the energy absorbed at each point within is  $T_0$  Mev/g-sec. The solid is everywhere traversed by an equilibrium electron flux  $I_z(T_0, T)$  electrons/cm<sup>2</sup>-sec-Mev, the spectrum of which is characteristic of the atomic number  $Z$  of the material. This same flux traverses the cavity, which is assumed not to perturb the spectrum. Each electron crossing the cavity is considered as losing energy continuously (in infinitesimal steps) at a rate  ${}_m S_a(T)$  Mev-cm<sup>2</sup>/g. Thus the energy dissipated per gram of air divided by that per gram of wall material is given by the ratio

$$f_z(T_0) = \frac{1}{T_0} \int_0^{T_0} I_z(T_0, T) {}_m S_a(T) dT. \quad (9)$$

If now only continuous energy losses are allowed in the wall material also, we can write

<sup>6</sup> Note again that the phrases "absorbed energy" and "energy lost by the traversing electrons" can be used interchangeably so long as delta-ray production is ignored; see sections 2.3.a and 2.3.b.

$$I_z(T_0, T) = \frac{1}{{}_m S_z(T)}, \quad (10)$$

bearing in mind for the sake of dimensional equality the fact that one electron per second is generated in each gram of material. Then

$$f_z(T_0) = \frac{1}{T_0} \int_0^{T_0} \frac{{}_m S_a(T)}{{}_m S_z(T)} dT. \quad (11)$$

If we assume, with Gray, that ratios of stopping powers are independent of energy and take the density ratio into account, this reduces to his expression for  $s^{-1}$ .

When there is a spectrum of initial electron energies,  $\frac{d\mu}{dT_0}$ , equation (9) must be replaced by

$$\bar{f}_z(T_0) = \frac{\int_0^{T_{max}} \frac{d\mu}{dT_0} \int_0^{T_0} I_z(T_0, T) {}_m S_a(T) dT dT_0}{\int_0^{T_{max}} \frac{d\mu}{dT_0} T_0 dT_0}, \quad (12)$$

for the ratio of the total energy absorbed per gram in the cavity to that in the wall. This evidently is equivalent to equation (5) from Laurence.

#### c. Modified Theory by Spencer and Attix

It has been stressed in the foregoing sections that all the previous derivations inherently require continuous energy losses by the electrons, both while they are in the cavity and while in the surrounding wall material. Allowing for the production of fast secondaries requires a basic change in approach to the problem. Such secondaries are produced in the cavity, carrying energy out, and are also produced in the wall material, thus modifying the spectrum of the electron flux traversing the cavity. A rigorous calculation would involve the complete analysis of the energy dissipated in the cavity, including

- a. Energy spent by electrons entering the cavity with insufficient range to span it.
- b. Energy spent by cavity-traversing electrons via production of secondaries incapable of reaching the cavity wall.
- c. Energy spent by fast secondaries originating in the cavity with sufficient energy to escape, and
- d. Energy spent by primary electrons generated by direct gamma ray interactions within the cavity.

It would be necessary in this analysis to consider the effect of the cavity shape as well as the configuration of the electron tracks under the influence of nuclear scattering as well as electron collisions. As Burch (1955) pointed out, such a rigorous treatment appears to present nearly insurmountable mathematical difficulties. Furthermore, the cross sections for production of low energy secondaries are not well enough known for this purpose. Spencer and Attix (1955) reduced the degree of rigor just to the point where a numerical calculation seemed feasible, at the risk of oversimplifying the physical picture. First of all, the nuclear scattering, the configuration of the electron paths, and the cavity shape are not explicitly dealt with. Neither are primary gamma ray interactions. Other assumptions will emerge as the following dual-volume model is described.

Consider a small solid-filled cavity  $V_2$  in an extended region of the same material under uniform gamma radiation.  $V_2$  need not be infinitesimal, but must be "small" in a sense to be defined presently. Its shape is purposely vague, but its average diameter is characterized by  $\Delta$ , the energy needed by an electron to be able to just cross it before stopping. We wish to write an expression for the energy dissipated per gram within  $V_2$  in terms of the equilibrium electron flux traversing it. This dissipated energy will be made up of the contributions described in "a," "b," and "c" above for which some simpler picture must be substituted to allow calculation of the result.

The following two-group picture was decided upon: all secondaries originating with energies less than  $\Delta$  and other electrons falling below  $\Delta$  in energy are called "slow" and are assumed to dissipate their energy on the spot where they originate or become "slow." All electrons with starting energies greater than  $\Delta$  carry their energy elsewhere and can thus be regarded as part of the "fast" electron flux.<sup>7</sup> The electron removed in what is usually thought of as an ionization event is here regarded as a slow secondary.

The consequences of this are: First, that no energy is allowed to be brought into  $V_2$  by slow electrons, hence the contribution under "a" above is taken as zero, obviously an underestimate. Second, that the contribution from "b" consists of the total energy of all secondaries produced in  $V_2$  with original energy less than  $\Delta$ , clearly an overestimate as some of these will leave the cavity. And third, that the contributions from "c" come only from secondaries with starting energies greater than  $\Delta$ , and hence must be zero,

<sup>7</sup>Note that under the continuous-loss assumption all secondaries are regarded as dropping their energies "on the spot."

again an underestimate. The net result is probably an underestimate of the energy spent in  $V_2$  as the errors in "a" and "b" alone should compensate one another exactly in  $V_2$  (although only approximately in a cavity containing a material of different atomic number than the wall). The setting of "c" and "d" equal to zero thus causes a deficit.

Based on this model one can write for the approximate energy absorbed (Mev/g-sec) in the wall material

$$T_0 = \int_{\Delta}^{T_0} I_z(T_0, T) {}_mS_z(T, \Delta) dT, \quad (13)$$

where the uniform gamma ray field is again assumed to release everywhere one electron per gram per second of energy  $T_0$ .  $I_z(T_0, T)$  is the "fast" flux ( $T > \Delta$ ) of primary plus secondary electrons traversing the cavity.  ${}_mS_z(T, \Delta)$  is a modified stopping power, based on the Møller formula, in which only those interactions generating a secondary of  $T < \Delta$  are counted. Whichever electron has the lesser energy after an interaction is to be regarded as the secondary. The integral is given the lower limit  $\Delta$  since  $I_z(T_0, T)$  is taken as zero for  $T < \Delta$ . Spencer and Fano (1954) express  $I_z(T_0, T)$  by the product  $R_z(T_0, T) \times ({}_mS_z(T))^{-1}$ , where  $R_z(T_0, T)$  is the ratio of the total electron flux to the flux of primary electrons alone.

Equation (13) can be viewed as an approximate expression of the familiar equilibrium conditions. The left side is the energy released by gamma rays and is precisely the energy absorbed. The right side is the energy absorbed from traversing electrons based on the above schematization.

We must require  $\Delta \ll T_0$  partly for the reason that, were this not so, the assumption that the direct gamma ray interactions "d" can be neglected would not be valid and equation (13) would not hold.

Consider now a second cavity of identical  $\Delta$ , but filled with air instead of the solid material. The energy dissipated (Mev/g-sec) in this cavity will be given by

$$\int_{\Delta}^{T_0} I_z(T_0, T) {}_mS_a(T, \Delta) dT, \quad (14)$$

and the ratio of the energy absorbed in the air cavity to that in the solid cavity for primaries of energy  $T_0$  can be written as

$$f_z(T_0, \Delta) = \frac{\int_{\Delta}^{T_0} I_z(T_0, T) {}_mS_a(T, \Delta) dT}{\int_{\Delta}^{T_0} I_z(T_0, T) {}_mS_z(T, \Delta) dT} \quad (15)$$

This can be further extended over the whole spectrum of  $T_0$  giving an equation analogous to equation (12), with limits of integration  $\Delta$  to  $T_{max}$ .

It was mentioned previously, in the discussion of the two-group schematization for secondaries, that it probably results in an underestimate of the energy dissipation in a cavity. However, numerical computations of equation (13) for  $\Delta \ll T_0$  indicate that, with  $I_z(T_0, T)$  and  $S_z(T, \Delta)$  values as described further by Spencer and Attix (1955), the equality holds within about 1 percent. Furthermore, by expressing equation (15) as a ratio of similar terms, the errors tend to cancel out. The closer  $Z_z$  and  $Z_a$  are to each other, the better the approximation will be.

Note that  $f_z(T_0, \Delta)$  is a function of  $\Delta$ , and may be expected to vary with the cavity size or the gas pressure. Qualitatively, the physical explanation for this is as follows. If  $Z_z > Z_a$ , the ratio  ${}_mS_a(T, \Delta)/{}_mS_z(T, \Delta)$  increases with decreasing  $T$ . As  $\Delta$  (cavity size) decreases,  $I_z(T_0, T)$  contains electrons of lower and lower energies in numbers large enough to make their presence felt. Thus  $f_z(T_0, \Delta)$  tends to increase.

The theory does not predict in detail what  $f_z(T_0, \Delta)$  should do as  $\Delta$  approaches zero. This would depend upon the behavior of the functions  $I(T_0, T)$  and  ${}_mS(T, \Delta)$  as  $\Delta \rightarrow 0$ , which is complicated by atomic binding effects and is not known at present. There is some experimental evidence (Larson, 1956) (see section 4.1.) indicating that  $f_z(T_0, \Delta)$  should finally approach a constant slope for very small  $\Delta$ , but this behavior is not exhibited by the calculated  $f_z(T_0, \Delta)$  for  $\Delta$  as low as 2 kev.

In application to a chamber of variable size  $f_z(T_0, \Delta)$  is to be calculated and applied for each  $\Delta$ -setting of the chamber size, where always  $\Delta \ll T_0$ , say  $\Delta < 0.1 T_0$  at most. The resulting plot of corrected ionization per gram of air versus  $\Delta$  should be more nearly flat than the original uncorrected curve. An extrapolation to zero volume is then made possible.

#### d. Modified Theory by Burch

Burch (1955, 1957) used the familiar dual-volume model with the usual requirement of gamma ray field uniformity. In his initial paper (1955) only infinitesimal<sup>8</sup> cavities were considered and attention was focused primarily on spherically shaped ones for the sake of simplicity.

<sup>8</sup> By this Burch does not mean infinitesimal in a mathematical sense. The cavities are large enough to contain macroscopic portions of electron tracks and to allow for production of secondaries which may or may not reach the wall. They are said to be infinitesimal enough, though, to avoid the necessity of dealing with (a) electrons entering the cavity with insufficient energy to cross, or (b) direct gamma ray interactions.

The cavities are related in their sizes (both being infinitesimal) by

$$l_{T,a} \left( \frac{dT}{dx} \right)_a = l_{T,z} \left( \frac{dT}{dx} \right)_z, \quad (16)$$

where  $l_{T,a}$  and  $l_{T,z}$  are the average path lengths within the air or solid cavities, respectively, of electrons with energies between  $T$  and  $T+dT$ .  $(dT/dx)$  is the average energy they dissipate per unit distance along their track within either cavity. Energy carried out of the cavity by secondaries or by radiative processes is not counted in  $dT/dx$ , which is thus a function of cavity size and shape.  $(dT/\rho dx)$  corresponds in meaning to the term  ${}_mS(T, \Delta)$  used by Spencer and Attix.  $\rho$  is the density in g/cm<sup>3</sup>.

As an approximation, Burch defines as completely dissipative any collision within the cavity resulting in the production of a secondary of energy  $\eta$  less than the average energy,  $\eta_a$  or  $\eta_z$ , needed to reach the cavity wall. Secondaries having energies greater than  $\eta_a$  or  $\eta_z$  are taken to be completely nondissipative. This differs from the Spencer-Attix assumption in two respects.

In the first place, they take  $\Delta$  as the energy needed on the average to *span* the cavity rather than to *escape* it from the inside. As was discussed in section 2.4.c, this is clearly an underestimate but is needed to partially offset other known underestimates in the energy dissipated in the cavity. Specifically, in the Spencer-Attix schematization, electrons with energies less than  $\Delta$  are not allowed to enter the cavity nor to leave it. Actually they do both and tend to compensate one another. Burch (1957) has referred to this omission of what he calls "tail-ends" of tracks (for "noninfinitesimal" cavities) and to the overestimate of  $\Delta$  as being two separate errors. Rather they are complementary assumptions. In Burch's picture, on the other hand, an electron of energy less than  $\eta_a$  or  $\eta_z$  is allowed to enter and dissipate, but a corresponding secondary electron starting inside the cavity and carrying some of its energy out will also be counted as fully dissipative so that the two cannot balance one another and a surplus results. His discounting of dissipated energy by all secondaries of  $\eta > \eta_a, \eta_z$ , however, throws away more energy of the kind described under "c" in section 2.4.c than does the corresponding assumption by Spencer and Attix, since  $\Delta > \eta_a, \eta_z$ . Hence there is again some qualitative compensation.

The energy balance situation in the Burch cavities is thus found to be qualitatively very similar to that discussed in section 2.4.c. Neither treatment rigorously accounts for

all the energy dissipated in a cavity, and in the final analysis both rely on the *ratio* of energies spent in the two cavities being insensitive to similar errors applying to both cavities.

A second, and more important, difference between the two models is the fact that Burch uses a different cutoff energy,  $\eta_a$ , in the air cavity than that,  $\eta_z$ , in the solid cavity. Spencer and Attix use the same  $\Delta$  for both. In fact, the failure of Burch's theory to reach the point of full numerical solution was ascribed by him to the immense difficulties encountered in deriving the relationship between  $\eta_a$  and  $\eta_z$ . It is easy to show that Burch is, strictly speaking, correct in this requirement. Equation (16) relates the two cavity dimensions by the ratio of the energy-dissipation powers at energy  $T$ . At much lower energies,  $T \approx \eta_a, \eta_z$ , the same ratio will not hold strictly, so that the average energy  $\eta_a$  needed by a secondary to reach the wall in the air cavity will in general differ from  $\eta_z$ . If  $Z_z > Z_a$ , and ignoring nuclear scattering,  $\eta_a$  would be greater than  $\eta_z$ . Nuclear scattering, however, would make the path of an electron more circuitous in the solid. This effect will be more pronounced for secondaries than for the higher energy electrons crossing the cavity, tending to make  $\eta_z > \eta_a$ . Thus the two effects tend to compensate, but it would be difficult to say how well. The assumption  $\eta_a = \eta_z$  would seem to be the best one can do to achieve a numerical solution at present.

Burch's derivation proceeds along the lines used by Gray, except for the substitution of dissipative for the total stopping power. He defines a term

$$R_T = \frac{\rho_a \left( \frac{dT}{dx} \right)_z}{\rho_z \left( \frac{dT}{dx} \right)_a}, \quad (17)$$

as the mass energy dissipation ratio for traversing electrons of energies between  $T$  and  $T+dT$  (analogous to Gray's  $s$  multiplied by the density ratio). This is shown to be the ratio of energy absorbed per gram of solid to that in air for electrons between  $T$  and  $T+dT$ . This is integrated over  $T$  from 0 to  $T_0$  to obtain the average value  $\bar{R}$ .

$$\bar{R} = \frac{\int_0^{T_0} R_T n_{T,a} l_{T,a} \left( \frac{dT}{dx} \right)_a dT}{\int_0^{T_0} n_{T,a} l_{T,a} \left( \frac{dT}{dx} \right)_a dT} \quad (18)$$

Here  $n_{T,a} dT$  is the number of electrons crossing the cavity during the irradiation having a kinetic energy (at the cavity) in the range  $T$  to  $T+dT$ .  $n_{T,a} l_{T,a} \left( \frac{dT}{dx} \right)_a dT$  can be seen to be a weighting factor for  $R_T$  that depends on the spectrum of the incoming flux of electrons and on cavity shape and size effects.

Burch carried out one pilot calculation for a 2 cm diameter spherical graphite wall air cavity for  $\text{Co}^{60}$  and  $\text{Na}^{24}$  gamma rays. He initially assumed  $\eta_a = \eta_z = 40$  kev and then corrected the resulting  $\bar{R}$ , equal to the ratio of ionization in a free-air chamber to that in the cavity, downward by 0.5 percent on the basis of an estimated difference between  $\eta_a$  and  $\eta_z$ . For  $\text{Co}^{60}$  this procedure resulted in an ionization ratio  $J_{\text{air}}/J_{\text{graphite}}$  of 0.994, indicating that a graphite walled cavity should yield more ionization than free air for  $\text{Co}^{60}$  gamma rays. This effect should become apparent in experiments where the air pressure is varied in a graphite cavity; i.e., the ionization per unit pressure should increase slightly as the pressure is reduced. The reverse was actually found to be the case (Whyte, 1957; Attix, 1957) (section 5.1.). Calculations based upon the Spencer-Attix formulation agree with these experiments. Burch's result would more nearly agree with these if the difference between  $\eta_a$  and  $\eta_z$  were neglected. This indicates that this difference is needed for compensating for some other unbalance.

## 2.7. Other Considerations

### a. Electronic Equilibrium Requirement<sup>9</sup>

In each derivation uniform gamma radiation was required over a region of material large enough to produce complete electronic equilibrium conditions at the site of the cavity.<sup>10</sup> This requirement is necessary only if the *primary* electron spectrum is to be characterized by the reciprocal of the stopping power,  $1/S(T)$ . There is no fundamental reason why equation (15), for example, could not be used with any type of  $I_z(T_0, T)$  so long as it could be identified and was sensibly constant over the dimensions of the cavity itself. Thus cavity ionization measurements should be meaningful if the electron flux does not vary appreciably over the cavity itself, even though the flux of primary gamma rays or neutrons might vary greatly in intensity over the region

<sup>9</sup> See also section 2.3.

<sup>10</sup> Complete electronic equilibrium will never exist in the cavity itself unless  $Z_a = Z_z$ , as can be seen from equations (13) and (15). This is because, under complete equilibrium, the energy carried out of a volume by electrons generated by gamma ray interactions within it must be just balanced by energy spent within the volume by electrons generated elsewhere. Equation (13) expresses this relation, while equation (15) shows that the relation does not hold where the cavity material differs from its surroundings.

in which the cavity-traversing electrons originate. The difficulty then arises in evaluating the spectrum of the (non-equilibrium) electron flux crossing the cavity. Such difficulties can be expected for X or gamma rays above a few Mev because such rays will be appreciably attenuated over the region in which the cavity-traversing electrons originate. If a beam of beta rays or of charged particles is incident on a chamber from the outside, there will usually be a nonequilibrium flux at the cavity.

#### b. Polarization Effect

Polarization can be treated as a perturbation on the stopping power formulas used in evaluating the Bragg-Gray relation (Whyte, 1954). It alters the high-energy flux entering the cavity and the energy dissipation within a solid-filled cavity, but it is not appreciable in a gas. It tends to decrease the stopping power of a solid below that for a gas of the same  $Z$ , and it is not smoothly  $Z$  dependent.<sup>11</sup>

### 3. Ranges and Stopping Powers

In section 2 it is shown that in Gray's approximation the  $s$  in the Bragg-Gray formula is the relative stopping power of the wall and the gas in the cavity. In more refined approximations  $s$  is still a function of the stopping powers of the materials. This makes it necessary to review our knowledge of these stopping powers. Only the stopping powers of electrons are needed for the analysis of cavity chamber data to be done later. The value of the mean excitation potential that appears in the electron stopping power equation must be found experimentally, however, and this is best done through experiments with heavy charged particles. This requires that the stopping powers of the latter be reviewed also.

The subject of stopping powers is very important to modern physics. The theory itself has been of fundamental importance in developing atomic models. The resulting

<sup>11</sup> The paper by Spencer and Attix (1955) contains an error in the treatment of the polarization effect. It is there included in its effect upon the incoming flux but not in the effect on energy dissipation within the solid filled cavity. Equation 3b in that paper should be altered to read

$$f_s(T_0, \Delta) = \frac{(Z/A)_{\text{air}}}{(Z/A)_s} \left\{ 1 + \frac{1}{T_0} \int_0^{T_0} R_s(T_0, T) \left[ \frac{B_{\text{air}}(T)}{B_s(T)} - 1 \right] dT + \frac{\Delta R_s(T_0, \Delta)}{T_0} \left[ \frac{B_{\text{air}}(\Delta)}{B_s(\Delta)} - 1 \right] \right\}$$

and the values given in table II changed correspondingly. A corrected table has been prepared in an errata sheet sent out with reprints.

range-energy curves and stopping-power-energy curves are used to determine energies and masses and to make corrections in nuclear cross section measurements. Because of its importance the subject has been reviewed fairly often. Bethe and Ashkin (1953) prepared a definitive review of the entire subject covering the literature up to December 31, 1951. The subject of stopping of heavy particles was reviewed by Allison and Warshaw (1953) covering the literature up to June 1953 and later by Uehling (1954) covering the literature to April 1954. The present report will use these reviews as a foundation to bring the subject up to date, especially in those aspects related to dosimetry. In general, references will be limited to those appearing since these reviews.

In September 1958, at Gatlinburg, Tenn., the National Academy of Sciences-National Research Council held a conference on the penetration of charged particles through matter. The latest information and evaluations of range and stopping power data were available at the conference. This material was used in the preparation of the present report. Proceedings of the conference will be published later.<sup>11a</sup> In the meantime, the present authors report their impressions of the data and views of the speakers at the conference.<sup>12</sup> The special reference notation (Gatlinburg, 1958) will be used for information obtained in this way.

#### 3.1. Theoretical Formulas for Stopping Powers

Charged particles heavier than electrons passing through matter lose energy principally by inelastic collisions with the atomic electrons of the material. The energy lost in individual encounters is so small that on a macroscopic scale the particle seems to lose energy continuously. The main changes in direction are due to relatively infrequent scattering by nuclei with little energy loss; therefore, the theory concerns itself only with the energy loss to the atomic electrons. The probability that a passing particle will raise an atom to an excited state can be calculated using Born's approximation. The energy lost, multiplied by the probability of the loss, and summed over all possible energy losses times the atomic density gives the stopping power  $S = -dT/dx$ . The resulting formula for the mass stopping power of charged

<sup>11a</sup> Nuclear Science Series, Report 29, National Academy of Sciences-National Research Council, 1960.

<sup>12</sup> The authors wish to acknowledge the private advice of several members of the conference on matters of range and stopping power.

particles heavier than electrons in a material of atomic number  $Z$  and atomic weight  $A$  is

$${}_mS = \frac{4\pi e^4 NZ z^2}{A m_0 v^2} \left[ \ln \frac{2 m_0 v^2}{(1-\beta^2)I} - \beta^2 - \frac{\sum_i C_i}{Z} - \frac{\delta}{2} \right], \quad (19)$$

$e$  is the charge on the electron,  $z$  is the number of electronic charges on the heavy particle,  $m_0$  is the rest mass of the electron, and  $v = \beta c$  is the velocity of the charged particle.  $\delta$  is a polarization correction.  $I$  is the mean excitation potential.

The  $C_i$  (usually written  $C_K$ ,  $C_L$ , etc., to denote the electron shell) are correction terms. The  $C_i$  are large when the particle velocity is comparable with the velocity of the electrons in the  $i$ th shell; at higher energies they decrease in inverse proportion to the energy. The formula for  ${}_mS$  without the  $C_i$  terms can be derived with Born's approximation and the limitation to velocities  $v \gg Ze^2/\hbar$ . The  $C_i$  corrections extend the validity to lower velocities. At still lower velocities the Born approximation requires modification. Furthermore, at very low velocities the probability of the charged particle capturing an electron is appreciable; no attempt is made in the present formulas to correct for the capture process.

$\delta$  is a correction for the polarization of the medium brought about by the electric field of the charged particle. The total polarization correction contains an energy independent term that depends on the electron density of the medium. It is customary to incorporate this constant term in the quantity  $I$ . Then  $\delta$  is zero for  $\beta < 1/\sqrt{\epsilon}$  where  $\epsilon$  is the static dielectric constant.

$I$  is called the mean excitation potential of the medium. Theoretically it depends only on the nature of the medium and not on the velocity or the type of charged particle. It may depend on the electron density of the medium because of the combination with the polarization correction just mentioned. Bloch (1933) deduced that for a medium of free atoms  $I$  should be proportional to the atomic number; i.e.,  $I = kZ$ . Except for a factor to account for the polarization effect,  $I$  is the geometric mean of the average excitation potentials of all shells in the atom weighted by the oscillator strengths of the shells. Unfortunately this information is generally not available, theoretically or experimentally, so  $I$  cannot be calculated. In general, the values of  $I$  must be found by fitting equation (19) to experimental data for stopping powers or ranges.

In application of the theory to electrons it is necessary to take into account the indistinguishability of the incident electron and the atomic electron after their interaction. It is conventional to identify the electron with the most energy after the interaction as the primary one. This means that an electron can lose up to half its energy in a single interaction. There will be a much larger spread ("straggling") in the energy losses about the average value than in the case of heavy particles, but the concept of an average is still a useful one. Electrons are also much more strongly deflected by collisions with nuclei than heavy particles. The theory gives the rate of energy loss along the actual path of the electron rather than along a line in the direction of incidence. The resulting formula for the mass stopping power of electrons due to ionization and excitation of atomic electrons of the stopping medium is

$${}_mS = \frac{2\pi e^4 NZ}{A m_0 v^2} \left[ \ln \frac{m_0 v^2 T}{4(1-\beta^2)I^2} + 1 - \beta^2 + \left(\frac{T}{W}\right)^2 \left(\frac{1}{S} + \ln 2\right) - \delta \right]. \quad (20)$$

$W$  is the total energy of the electron;  $W = T + m_0 c^2$ . The other symbols are the same as above. In particular, the same value of  $I$  is expected to apply to both heavy particles and to electrons. There is no theory for  $C_i$  corrections for electrons. Fortunately the corrections would be small except for very low energy electrons.

Positrons can be distinguished from the atomic electrons after their interaction so it is necessary to allow for the possibility that the positron may lose all its energy in a single collision. Furthermore, the positron-electron cross section differs from the electron-electron cross section for large energy transfers.

$${}_mS = \frac{2\pi e^4 NZ}{A m_0 v^2} \left[ \ln \frac{2 m_0 v^2 T}{(1-\beta^2)I^2} - \frac{\beta^2}{12} \left\{ 23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3} \right\} - \delta \right]. \quad (21)$$

In this equation,  $\tau = T/m_0 c^2$ .

### 3.2. Comparison of Theory and Experiment

The methods of studying the penetration of charged particles may be divided into roughly three classes: (1) Those in which the loss of energy is measured for layers that are a

small fraction of a range thick. Such measurements are direct tests of the stopping power formulas. Scattering corrections in this type of measurement are small. The loss of energy is small and therefore difficult to measure. At low energies thin layers are required for solid or liquid absorbers and it is difficult to make them uniform and to determine their thickness. (2) Those in which the energy loss is measured in layers that are a major fraction of a range thick but not equal to the total range. The stopping power must be integrated to give a range figure for comparison.

$$\Delta R = \int_{T_1}^{T_2} \frac{dT}{S} \quad (22)$$

If the low energy limit,  $T_1$ , is high enough, the  $C_i$  corrections will be small and can be made accurately. Scattering corrections are important. Scattering for electrons is so large that measurements of this type are of value only for heavy particles. (3) Those in which the total range for a given energy is determined. Scattering is important. It is necessary to have almost complete knowledge of the  $C_i$  for accurate comparison. Actually a complete comparison is not possible in this way because the theory is not applicable at very low energies. One must compute  $\Delta R$  from (22) above some low reference energy for which the range is reasonably well known and compare it with the observed range minus the reference range.

#### a. Heavy Particles

In general, at the time of the reviews referred to above the experimental data for heavy particles were considered to be in fairly good agreement with the theory. The work of Lindhard and Scharff (1953), however, particularly as interpreted by Allison and Warshaw (1953), suggested that  $I$  might be a function of the velocity of the particle instead of being velocity independent.

According to the theory of Lindhard and Scharff (1953), the stopping number per electron<sup>13</sup> of heavy particles, corrected for relativistic effects, is a function of the variable  $x = (\hbar v / e^2)^2 / Z$ .

$${}_e B' = {}_e B + \beta^2 + \ln(1 - \beta^2) = f(x) \quad (23)$$

<sup>13</sup> The stopping number per electron is defined as

$${}_e B = \frac{A m_0 e^2}{4\pi \rho N Z e^2} \left( -\frac{dT}{dx} \right)$$

Obtaining  ${}_e B$  from equation (19), but not including the  $C_i$  or  $\delta$ , and substituting it in (23) gives

$${}_e B' = \ln \frac{2 m_0 x^2}{I} = \ln \frac{4 R y}{k} x \quad (24)$$

$Ry = 13.60$  ev;  $k = I/Z$ . Data for  ${}_e B'$  for all elements plotted versus  $\log x$  should lie on a straight line with unit slope as long as the  $C_i$  corrections are small (i.e., as long as  $x \gg Z$ ) and  $I$  is independent of velocity. Lindhard and Scharff found that the available data did lie on a straight line for  $x$  greater than about 100. Below this value of  $x$  the data fell below the straight line (see fig. 1).

The first deviation from the straight line extrapolated from high energy data was due to the results of Sachs and Richardson (1951) for protons of 18 Mev. These results are now known to be in error. MacKenzie reported (Gatlinburg, 1958) that long after the experiment was completed, it was discovered that multiple scattering in the stopping foil had interfered with the energy analysis of the transmitted beam. Later experiments by Sonett and MacKenzie (1955) and Burkig and MacKenzie (1957) in the same energy region showed that the points for protons were on a straight line that was at least parallel to the line extrapolated from higher energies. This is shown in figure 1. Both experiments gave relative stopping powers, so normalization at one point was necessary; hence it could not be said that they were on the *same* straight line. An experiment of Brolley and Ribe (1955) with 4.43 Mev protons and 8.86 Mev deuterons gave a straight line parallel to the high energy extrapolation but displaced slightly from it (part of their measurements were absolute and were used to normalize the rest).

It has always been recognized that not all data should lie on a straight line on the Lindhard-Scharff plot. By convention actual experimental data are put on this plot without correction for the  $C_i$ . If the  $C_i$  are included, they account for a small part of the deviation from linearity found by Lindhard and Scharff. In the case of aluminum about one-third of the deviation could be due to the  $C_i$ . Uehling demonstrated (Gatlinburg, 1958) that, particularly for the light elements, most of the pertinent  $C_i$  corrections for a given atomic number but variable energy were approximately linear when placed on a Lindhard-Scharff type of scale. This causes the uncorrected experimental data to lie on straight lines having a slope different from unity and causes lines for different materials to be slightly displaced from one

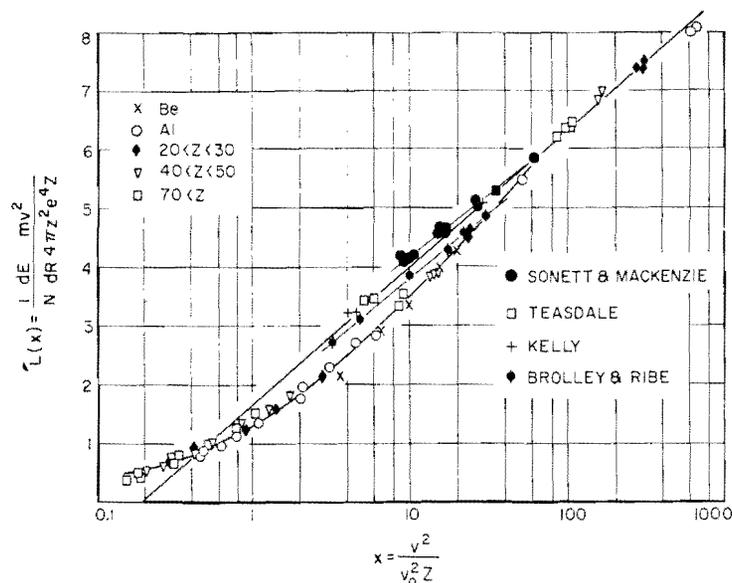


FIGURE 1. Lindhard-Scharff plot of stopping power data.

Data compiled by Lindhard and Scharff, to which have been added later data; points for Al, Au, and Th by Teasdale; and points for Al, Bi, and Th by Kelly. Along a line, which is parallel to a line through the present data, are shown the points for H<sub>2</sub>, He, N<sub>2</sub>, O<sub>2</sub>, Ne, Ar, Kr, and Xe obtained by Brolley and Ribe. ( $\alpha_0 = e^2/\hbar$ ). (Sonnnett and MacKenzie, 1955).

another. He also showed that data of different investigators for constant energy but different atomic numbers lay on straight lines with slope not equal to unity. He showed that plotting  $\epsilon B'$  versus a modified variable due to Brandt (see section 3.3.c),  $x' = (\hbar v/e^2)^2/Z(1+aZ^{-2/3})$ , corrected the slopes to unity but left lines for different energies slightly displaced. The displacement is presumably due to the effects of the  $C_L$ .

Other experiments at even lower energies also show that  $I$  does not vary with energy.  $I$  calculated to fit the experimental data of Kahu (1953) is constant above  $x=2.5$  for aluminum, 2.2 for copper, and 0.8 for gold. Below these points the  $C_L$  corrections that were not made would become important and finally the theory would be inapplicable because of the assumptions on which it is based. Reynolds, Dunbar, Wentzel, and Whaling (1953) showed similarly that  $I$  for low  $Z$  gases is nearly velocity independent at  $x=3$ .

Sachs and Richardson (1953) pointed out the possible existence of another type of variation of  $I$  with proton energy above 10 Mev for aluminum. This conjecture was based on experimental data that showed a logarithmic decrease in  $I$  with increasing proton energy. Caldwell (1955) made a new computation of the  $C_L$  corrections for these data and showed that with the exception of the  $I$  value measured at about 300 Mev by Mather and Segrè (1951), the data were consistent with  $I$  being independent of velocity. There are four sets of data at high energies with which Mather and Segrè's may be compared, Bakker and Segrè (1951), Thompson (1952), Zrelov and Stoletov (1959), and Barkas and von Friessen (1959, 1961). The first two were both relative  $S$  measurements and the interpretation depends on what is taken as the reference. If the  $I$  values obtained by Bakker and Segrè are normalized so  $I_{Al}=150$  ev (this also fixes Thompson's results since he referred to the Bakker-Segrè value for copper), which is close to Mather and Segrè's result, then there is poor agreement among the  $I_{Cu}$  and  $I_{Pb}$  values (table 1). If the Bakker-Segrè values are normalized so  $I_{Al}=164$  to agree with the low energy results (see sec. 3.3.a and table 2), then the  $I_{Cu}$  and  $I_{Pb}$  values are in excellent agreement but Bakker-Segrè's  $I_C$  is high. Thompson demonstrated that it is very difficult to measure stopping power in graphite because the result depends on the orientation of the sample. Zrelov and Stoletov made an absolute measurement for copper with 660-Mev protons, using a technique similar to that of Mather and Segrè. They found an ionization potential of 305 ev, which is in good agreement with Mather and Segrè's 310 ev. The agreement between the two groups of investigators strongly suggests that we accept their value for copper, in which case we must accept the re-normalization of the Bakker-Segrè results described above, unless we say that the relative values of Bakker and Segrè are not right. However, the latter have recently been confirmed by Barkas and von Friessen (1959, 1961) who made relative stopping power measurements with 750 Mev protons. They agree with the general shape of  $I/Z$  found by Bakker and Segrè. Using for the purpose of normalization an ionization potential of 163 ev for aluminum, and assuming that at 750 Mev the only tight-binding corrections required are those for the  $K$  and  $L$  shells, Barkas and von Friessen find the following ionization potentials: copper, 323; lead, 826; uranium, 917; and emulsion, 328. On the basis of all the evidence discussed above we conclude that there is probably no variation of  $I$  with proton velocity.

### b. Electrons: The Polarization Correction

Measurements of range or stopping power for electrons are hard to interpret in terms of equations (20) and (21) because of the strong scattering of the electrons and because the energy loss in individual collisions is subject to such wide statistical variation that determination of average values is difficult (the latter effect is referred to as "energy straggling"). In addition, electrons lose energy by radiation, and such losses are very important at high energies. Radiation losses are not included in (20) or (21); formulas for the radiation losses are available (Bethe, 1953). Furthermore, high energy electrons have velocities high enough that the velocity dependent part of the polarization correction is appreciable. Several recent experiments have further attested to the correctness of the theoretical stopping power formulas for electrons and to the accuracy of the calculation of the polarization correction.

Hudson (1957) made very precise measurements of the energy loss of 150-Mev electrons in thin layers of lithium, beryllium, carbon, and aluminum. The results were in agreement with the theory to within 2 percent. The polarization corrections were taken from Sternheimer (1952, 1956). Sternheimer made two sets of calculations, one on the basis of  $I$  values from Bakker and Segrè (1951) and one on the basis of values from Caldwell (1955). The differences are not large, but Hudson's results were in better agreement with the latter.

Goldwasser, Mills, and Robillard (1955) used 15.7-Mev electrons to show that the difference in stopping powers for solid and gaseous teflon and Kel-F due to the dependence of the polarization correction on density were correctly given by Sternheimer. Barber (1956) demonstrated that the energy and density dependence of the polarization effect in gases for 2 to 35 Mev electrons was in fair agreement with Sternheimer's calculations.

### 3.3. The Mean Excitation Potentials

A simple way of summarizing knowledge about stopping powers is to give the value of the mean excitation potential  $I$ . There is difficulty in determining  $I$  accurately. Measurements of stopping powers or of ranges determine  $\ln I$  rather than  $I$ . As a result the relative errors in the  $I$  values are five to ten times those in the measured stopping power or range. Conversely, of course, the  $I$  values do not have to be known as accurately to get accurate values for the stopping power.

For X- and gamma ray, beta ray, and electron beam dosimetry, one is dealing only with relatively high velocity electrons. There are no  $C_i$  corrections to worry about. In section 4 the cavity chamber formulas for  $s$  will be put in such a form that one need only know the  $I$  values of the wall and gas to evaluate  $s$ .

Since  $S$  must be known quite accurately to obtain a good value for  $I$ , experiments with electrons are not employed for this purpose because of the large corrections necessary for straggling, scattering, and radiation losses. These complications in the passage of electrons through matter are discussed by Birkhoff (1958). Recently experiments were begun in which electron stopping powers are measured calorimetrically (Kalil et al., 1959; Ziemer et al., 1959). These show considerable promise because the stopping foil can be made thin enough to minimize the corrections needed while the electron beam intensity can be made large enough to give an easily measurable temperature rise in the foil. Until better data for electrons become available, data for heavy particles such as protons, deuterons, and alpha particles are used in the determination of  $I$ .

### a. The Elements

Table 1 presents measured  $I$  values for the elements reported since 1950. Entries in parentheses are relative values determined by assuming the value indicated by an asterisk for a standard substance.

When it is found that an  $I$  value,  $I_1$ , determined relative to  $I_{01}$  for some standard substance is in error because the value for the standard substance has been redetermined to be  $I_{02}$ , then the renormalized value,  $I_2$ , can be found from

$$I_2 = I_1 \left( \frac{I_{02}}{I_{01}} \right) \left( \frac{Z}{A} \right)_0 \left( \frac{A}{Z} \right)_{mS_0} \quad (25)$$

Aluminum and copper are the standard substances usually used in relative measurements. The  $I$  value for aluminum is now quite accurately known. When the result of Mather and Segrè is rejected as discussed above, the average of the absolute measurements listed in table 1 gives  $I_{Al} = 163$  ev. The value obtained by Sachs and Richardson (1953; Caldwell, 1955) was included in this average although, in general, their results are considered erroneous (section 3.2.a). MacKenzie (Gatlinburg, 1958) said that Sachs and Richardson's stopping power for aluminum fell very close

TABLE 1. Recent measurement of  $I$  (ev)

Author	Proton Energy, Mev	Method <sup>c</sup>	Reynolds et al. (1953)	Allison, Wushaw (1953)	Kahn (1953)	Brolley, Ribe (1955)	Simmons (1952)	Bichsel et al. (1957)	Sachs, Richardson (1953), Caldwell, Well (1955)	Hubbard, MacKenzie (1957)	Burkig, MacKenzie (1957)	Bloembergen, van Heerden (1954)	Thompson (1952)	Bakker, Segrè (1951)	Mather, Segrè (1951)	Zrelav and Stoletoy (1959)	Barkas and von Przeworski (1959, 1961)
	>0.6	1	1	~1.7	1.6	4.43	10	6-18	18	18	19.8	35-115	270	300	0-340	0.658	600-750 450-600 300-450
1 H						19							(18, 2)				2, 3
2 He						43											(45)
3 Li																	
4 Be								63.4									
6 C						67											
7 N						84											
8 O						80											
9 F						100											
13 Al							155	105.45	163.1	170	166.4*	162		130*	118		163*
26 Fe									363		(328, 8)			(243)			(273)
28 Ni									375.6		(366.0)	370		(279)	310		303*
29 Cu									585		(587.0)			(428)			(468)
42 Ru									659					(479)			(480)
47 Ag									654					(607)			
48 Cd									708					(758)			
50 Sn									962					(681)			
73 Ta									1136								
74 W																	
79 Au																	
82 Pb																	
92 U																	

Values in parentheses are relative to a value for a standard substance indicated by \*.

<sup>a</sup> From the value of  $I/Z$  extrapolated to high energy.

<sup>b</sup>  $C_K$  from Walske (1952) substituted for those from Brown (1950).

<sup>c</sup> Calculated for this report.

<sup>d</sup> Part of the experiment performed with 8.86 Mev deuterons.

<sup>e</sup> The numbers refer to the three types of measurement described in section 3.2.

to the straight line extrapolation from high energy on the Lindhard-Scharff plot and that therefore it evidently did not suffer much from multiple scattering and could be accepted.

Bichsel (Gatlinburg, 1958) reported a new treatment of the data of Bichsel, Mozley, and Aron (1957) given in table 1. The values listed in the table were obtained with  $C_K$ ,  $C_L$ ,  $C_M$ , etc., corrections taken or estimated from the literature. Only the  $C_K$  correction is expected to be very accurate for aluminum this way. In the new treatment the  $C_K$  correction was applied to the data for aluminum and then an asymptotic form for  $C_L$  (proportional to  $T^{-1}$ ) was fitted to the data at the highest energies and a value for  $I_{Al}$  found. Then  $C_L$  at the low energies was calculated using this  $I_{Al}$ . Finally, the asymptotic expression was varied until the low energy  $C_L$  took on what appeared to be a physically reasonable form. The corresponding  $I_{Al}$  could not be fixed exactly but was between 163 and 164 ev. Bichsel preferred 164.

The results of Burkig and MacKenzie (1957) and Bakker and Segrè (1951) have been renormalized to  $I_{Al}=164$  ev. The renormalized values are given in table 2.

TABLE 2. Recent measurements of  $I$ —renormalized values \*

Element	Author		
	Burkig, MacKenzie (1957)	Thompson (1952)	Bakker, Segrè (1951)
1 H		20.7 ev	
3 Li			38 ev
4 Be	62 ev		67
6 C		78.4	84.0
7 N		85.1	
8 O		98.3	
13 Al	164*		164*
26 Fe	324		264
28 Cu	361	306*	303
47 Ag	580		462
50 Sn			517
74 W	911		750
79 Au	987		
82 Pb	1,000		814
92 U			945

\* The substance used as a reference in the renormalization is indicated by \*.

Thompson's measurements (1952) were relative to  $I_{Cu}=279$  ev taken from Bakker and Segrè. The renormalized value for Bakker and Segrè (303 ev) is in good agreement with the result of Mather and Segrè (310 ev) and of Zrelav and Stoletoy (305 ev). There also happens to be good agreement with

the low energy point of Kahn (313 ev), but in view of the absence of any  $C_L$  correction in this work the agreement cannot be considered significant. There is serious disagreement between these high energy values and the lower energy results of Bloembergen and van Heerden (370 ev) and Bichsel, Mozley, and Aron (375.6 ev). Bichsel (Gatlinburg, 1958) reported a tentative analysis of some new data for nickel. He obtained  $I_{Ni}=337$  ev if  $C_K+C_L$ , but no  $C_M$  corrections were applied. An estimated  $C_M$  correction led to a variable  $I_{Ni}$  of about 314 ev. Correcting these to copper by proportion to the atomic number gives the values 349 and 325 ev. These are closer to the high energy values, but are still significantly different. A reconciliation of these results requires a discussion of the variation of  $I/Z$  with  $Z$ . This will be done in section 3.3.d. The conclusion drawn from that discussion will be that we should accept the high-energy values.

Thompson's results have been renormalized to  $I_{Cu}=306$  ev, an average between the values of Bakker and Segrè and Mather and Segrè, and close to the recent results of Zrelow and Stoletov and of Barkas and von Friessen. The renormalized values are given in table 2.

Thompson's values for the mean excitation potentials of hydrogen, carbon, nitrogen, and oxygen, were increased about 12 percent by the renormalization. This increase is supported by the work of Phelps, Huebner, and Hutchinson (1954). They found that the stopping powers of thin organic films for alpha particles calculated from the original Thompson  $I$  values were too high by approximately 3 percent. The increase in Thompson's  $I$  values due to the renormalization gives agreement within the experimental errors of the measurements.

#### b. Bragg's Law

Some mean excitation potentials have been measured for compounds and homogeneous mixtures, but according to Bragg's law they can be obtained from the  $I$  values for the elements. Bragg's law assumes that the atoms of a material act independently and independent of molecular binding forces in the stopping of charged particles. Under these conditions the energy lost by a charged particle is the sum of the losses to the constituents considered separately. Then the stopping power is given by

$$S = \sum_i v_i S_i, \quad (26.a)$$

$${}_m S = \sum_i u_i m_i S_i. \quad (26.b)$$

In these equations the  $v_i$  are the fractions by volume and  $u_i$  are the fractions by weight of the  $i$ th element in the compound or mixture. Another way of expressing the Bragg law is to say that formulas (19), (20), and (21) hold for compounds and mixtures with proper average values used for the parameters that depend on the nature of the medium. The proper average values are

$$\frac{Z}{A} = \sum_i u_i \frac{Z_i}{A_i}, \quad (27.a)$$

$$\ln I = \left(\frac{Z}{A}\right)^{-1} \sum_i u_i \frac{Z_i}{A_i} \ln I_i, \quad (27.b)$$

$$\frac{C_K}{Z} = \left(\frac{Z}{A}\right)^{-1} \sum_i u_i \frac{C_{Ki}}{A_i}, \quad (27.c)$$

$$\delta = \left(\frac{Z}{A}\right)^{-1} \sum_i u_i \frac{Z_i \delta_i}{A_i}. \quad (27.d)$$

Since most of the electrons in an atom are unaffected by chemical and intermolecular forces, their contribution to the stopping power should be the same for compounds as for free atoms. The valence electrons of an atom are influenced by these forces and will contribute differently to the stopping. When the proportion of valence electrons is large, as it is in the light elements, the change in stopping power may be appreciable and Bragg's law may not hold. When the velocity of the charged particle is low, the inner electrons of an atom are less effective in stopping. This makes the effective relative number of valence electrons larger and hence accentuates the deviations from Bragg's law. It is necessary to determine experimentally how large the deviations from Bragg's law are at high energies due to the first of these effects and at what energy the second effect becomes important.

Gray (1944) found that in 38 alpha particle range measurements in 15 gaseous compounds of hydrogen, carbon, nitrogen, and oxygen, departures from the Bragg law exceeded 3 percent in only 3 cases and did not amount on the average to more than 1.5 percent. Reynolds et al. (1953) tested the law for 0.03 to 0.6 Mev protons in gases.  $H_2O$ ,  $NH_3$ , and  $N_2O$  followed the law for protons above 0.2 Mev. NO never followed it in the range tested (this was apparent in Gray's review also); the stopping power was about 4 percent higher than calculated from the data for nitrogen

and oxygen. This could be explained by the results of Thompson (below) if at least part of the changes he found were due to molecular binding effects.

Thompson (1952) gave the Bragg law a very precise test using protons of 270-Mev average energy. He found it to hold to about 1 percent. The largest deviations were for hydrogen and were about 2 percent. The deviations were negligible for chlorine (and, presumably, for heavier elements). He interpreted these small deviations as changes in the  $I$  values of the elements in the compounds due to molecular binding. As will become apparent below, some of the variation may also be due to differences in the polarizability of the substances. Renormalizing Thompson's results as described above gives the values listed in table 3 for different conditions of molecular binding.

TABLE 3.  $I$  Values for elements in compounds

THOMPSON (1952)		
Element	Molecular binding	$I$ ev
1 H	Saturated	17.6
	Unsaturated	14.8
6 C	Saturated	77.3
	Unsaturated	75.1
	Highly chlorinated	64.8
7 N	Amines, nitrates	99.5
	Ring	76.8
8 O	-O-	98.5
	O=	88.9
17 Cl	All	170

Westermarck (1954) was able to give a qualitative explanation for some of Thompson's results by comparing the change in  $I$  value with the change in chemical binding in compounds as reflected in changes in molar refraction.

Brandt (1958a) pointed out that some cases in which the Bragg law holds may result from a compensation of opposing effects. Combining atoms in a molecule in such a way that increased binding of the valence electrons causes an increase in the mean excitation potential results in a decrease in stopping power. This may be offset by a simultaneous decrease of the polarizability which decreases the polarization effect and increases the stopping power.

### c. Mean Excitation Potential of Air

The stopping power of air is very important in dosimetry. Most relative stopping powers are desired relative to air because it is the gas used in most ionization chambers.

Unfortunately there is not a great deal of data from which  $I$  for air can be derived.

The value of  $I_{\text{air}}=80.5$  ev found by Bethe (1937) has been used for several years. He determined this  $I$  by making the calculated difference in range between the alpha particles from ThC' and Po agree with experiment. A recalculation using the same experimental data but newer values of the fundamental constants and new  $C_K$  corrections by Walske (1952) gives  $I_{\text{air}}=85 \pm 1$  ev. The data of Brolley and Ribeiro (1955) for 4.43-Mev protons give  $I_{\text{air}}=85.5 \pm 1$  ev.

Other sources of information are not very useful. The results of Reynolds et al., given in table 1 can be combined according to the Bragg law, equation (27.b), to give 95 ev, but no  $C_L$  correction was made to their data and it would be significant for 0.6 Mev protons. Wilson (1941) measured the stopping power of aluminum relative to air for protons in the range 2 to 4 Mev. He calculated  $I_{\text{Al}}=150$  ev using Bethe's value, 80.5, for air. If instead we now accept  $I_{\text{Al}}=164$  ev, Wilson's data yield the value  $I_{\text{air}}=90 \pm 7$  ev.

The value adopted for this report is  $I_{\text{air}}=85$  ev.

Thompson's (1952) data for nitrogen and oxygen listed in table 2 (plus an estimated  $I_A=220$  ev based on proportion to atomic number) combine according to the Bragg law to give  $I_{\text{air}}=89 \pm 1$  ev, or about 5 percent higher than 85 ev. His measurements, however, were made in liquid oxygen and nitrogen. Sternheimer (1954) predicted that liquefied gases would have higher  $I$  values than the gaseous form due to the polarization effect that is combined with the mean excitation potential. The polarization effect should be negligible for the gases. He made some rough estimates that indicated oxygen should give 16 to 38 percent and nitrogen, 18 to 41 percent higher  $I$  values in the condensed state. Brandt (1956) used a better method of estimating the effect and calculated 5 and 4 percent increases for the two materials. This is good agreement with the observed increase. We can conclude that  $I_{\text{air}}=85$  ev for the gas, but that the materials of "air equivalent" ion chamber walls will have about 5 percent higher  $I$  values; i.e., about 1 percent higher stopping power.

There is independent evidence that the mean excitation potentials in gases and in solids or liquids are different. The measurements of Phelps et al., (1954) mentioned above support Thompson's  $I$  values for solids. Aniansson (1955) found stopping powers for alpha particles in solids that were an average of 3.3 percent less than those found by Gray (1944) for gases. Ellis, Rossi, and Failla (1952) found the relative mass stopping power of polystyrene and

acetylene was  $0.99 \pm 0.02$  and concluded that there was no difference between solid and gas. The 1 percent difference, if real, would indicate a 7 percent higher  $I$  value in the solid. It is possible that the effects of chemical binding are different between polystyrene and acetylene and tend to cancel the polarization effect. They also found (1955) that the relative stopping power of water in the liquid and vapor forms was  $1.00 \pm 0.05$ . A real difference of a few percent would have escaped them.

To summarize the discussion of the Bragg law: we can expect it to hold to better than 1 percent in stopping power in most cases involving the light elements and for  $\hbar v^2/e^2 > Z$  if we use the appropriate  $I$ 's for gases and for condensed media. In a few cases larger deviations will occur due to strong molecular binding forces. Table 3 can serve as a limited guide for anticipating these special cases. In heavy elements the Bragg law should hold to even better accuracy.

#### d. Interpolation Between $I$ values

Bloch (1933) applied stopping power theory to the Fermi-Thomas model of the atom and concluded that the mean excitation potential should be proportional to the atomic number,  $I = kZ$ . It was not possible to calculate the value of  $k$  theoretically. It had to be determined experimentally by fitting the observed values of  $S$ . The existence of such a constant would provide a means of interpolation between existing  $I$  value data to elements that have not yet been measured.

Table 4 shows  $I/Z$  for the experimentally determined  $I$  values listed in tables 1 and 2. Data rejected for various reasons in the preceding discussion were omitted from this table. As noted earlier the  $I$  values for aluminum are in good agreement at all energies. For our adopted value of  $I_{Al} = 164$  ev, we get  $I/Z = 12.6$  ev. Hydrogen and helium have  $I/Z$  values distinctly different from the other elements. They would be expected to be different because they are so elementary in structure that statistical averages that would result in regularities between atoms with more electrons would not apply to them. The value for beryllium is also much higher than for the other elements. This was predicted by Bohr (1949) and is due to the screening effect of conduction electrons. The effects of the low energy polarization effect between gases (data of Broilley and Ribe) and liquids or solids (data of Thompson) that was discussed above is readily apparent.

TABLE 4. Results for  $I/Z$  (ev)

Element	Author									
	Bradley, Ribe, 1955	Bichsel et al., 1957	Burkig, Macken- ziv, 1957a	Bloem- bergen, van Heerden, 1951	Thomp- son, 1952a	Bakker, Segré, 1951a	Mather, Segré, 1951	Zrelov, Stolev, 1959	Parkas, von Fri- sch, 1959, 1961	
1 H	19				20.7			15		
2 He	21.5									
3 Li						12.7				
4 Be		15.8				16.7				
5 B			13							
6 C	11.2				13.1					
7 N	12.0				12.2			12.4		
8 O	11.1				12.3					
9 Ne	8.9									
13 Al	12.8			12.2						12.5
18 A	10.6									
26 Fe			12.5			9.4		10.5		
29 Cu		12.9	12.4	12.8		10.4		10.5		
47 Ag		12.4	12.3			10.7				11.1
48 Cd						9.8				
50 Sn										9.8
50 Sb										
74 W						10.3				
79 Au		13.1				10.1				
82 Pb			12.9	10.9						
92 U						9.9		9.2		10.1
						10.3				10.1
Average for $Z$ greater than 13		12.8	12.5			10.0		10.3		10.4

\*  $I$ 's renormalized as described in section 3.3.3.

There is a clearcut discrepancy in the values of  $I/Z$  for elements heavier than aluminum between measurements at proton energies near 20 Mev and those near 300 Mev. The former are all close to an average value of 12.6 that is the same as that for aluminum. The latter are all close to an average value of 10.

Brandt (1956) made a theoretical study of this problem. He first pointed out that the regularity predicted by Bloch would only be expected to hold between isolated atoms. The polarization effect and the changes in binding energy of valence electrons when atoms combine into molecules and condense into liquids and solids may change the mean excitation potential significantly from the value for an isolated atom. Furthermore these changes will not be any regular function of atomic number. Finally, he recalled a more complete theoretical analysis of Jensen (1937) that led to the conclusion that

$$I_0 = kZ \left( 1 + \frac{a}{Z^{2/3}} \right). \quad (28)$$

$I_0$  is the mean excitation potential of the isolated atom. Jensen calculated an approximate value 0.77 for  $a$ . Brandt (Gatlinburg, 1958) obtained  $a=0.25$  by means of a variational calculation.

Brandt (1956) accepted the results of Bakker and Segrè and of Thompson after renormalization so  $I_{A1} = 165$  ev (an insignificant difference from our value, 164 ev). He then calculated corrections for polarization and valence effects in order to compute  $I_0$  from the observed mean excitation potentials. The resulting  $I_0$ 's were fitted to Jensen's relation and gave  $I_0 = 8.2 Z (1 + 0.7 Z^{-2/3})$ . (Brandt, 1958a, see fig. 2). There is considerable uncertainty in the value of  $a$ . The relation would be expected to apply to heavy elements where the statistical model of the atom would apply. Actually it seems to work well for all atomic numbers. One quite important consequence of this study was an explanation of the rather large difference in  $I/Z$  between aluminum and the heavier elements. It appears that aluminum is an exceptional material just like beryllium is. Brandt (Gatlinburg, 1958) estimated that the screening effect of conduction electrons should increase the mean excitation potential for aluminum 35 percent above the value for the isolated atom. The observed  $I_{A1}$  is this much greater than the  $I_0$  calculated from Brandt's fit of equation (28).

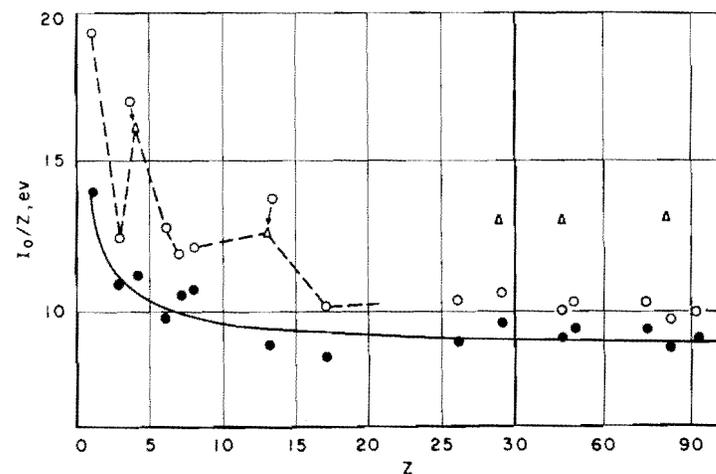


FIGURE 2. Reduced mean excitation potentials of isolated atoms.

Analysis according to Brandt (1956, 1958).

○  $I_0/Z$  of bound atoms, measured with 340-Mev protons (Bakker and Segrè, 1951), evaluated relative to  $I_{A1} = 165$  ev.

●  $I_0/Z$  of isolated atoms, calculated by Brandt.

△  $I_0/Z$  measured with 10 to 20 Mev protons (Caldwell, 1955; Richel, Mozley and Aron, 1957) and corrected for inner shells.

The solid curve represents a fit to Brandt's theoretical values, based on use of Jensen's formula (eq 28).

To explain the difference in  $I/Z$  for heavy elements between low and high energy experiments, Brandt (Gatlinburg, 1958) determined what total  $C_K + C_L + \dots$  correction would have to be applied to the experimental stopping powers of Burkig and MacKenzie in order to give mean excitation potentials that agreed with the high energy data. When these are compared with corrections obtained or estimated from the work of Walske (1952, 1956), the agreement is good for low atomic number but the Walske-type corrections are too low for high atomic number. Walske's calculations were based on the use of hydrogen-like wave functions for the atomic electrons. This should be a good approximation for the electrons of the innermost shells of an atom. For the heavy elements, the  $C_M$  and higher shell corrections may be higher than those due to the inner shells. The hydrogenic wave function approximation is not expected to be applicable to these electrons. Brandt employed an approximate method of calculating the  $C_1$  that is due to Lindhard and Scharff. The method is a statistical one that ignores the

properties of individual electron shells and considers only average electron behavior. The results of this calculation were in good agreement with the corrections necessary to resolve the disagreement between the low and high energy sets of data.

The solution to the problem of interpolating between measured  $I$  values for elements that have not been measured is: first calculate  $I_0$  from (28). Then calculate the polarization and valence corrections (Sternheimer, 1952 and 1956; Brandt, 1956) to obtain  $I$ . For practical purposes it will often be necessary to assume  $I=kZ$  and interpolate between the values for the high energy measurements in table 4. At worst this latter procedure should not result in more than a few percent error in a stopping power. Table 3 can serve as a guide for estimating valence and polarization effects.

#### e. Selected $I$ Values

When evaluating the parameters that enter into cavity ionization chamber theory, it is desirable to use consistently one set of values of the ionization potential. The preceding review indicates that there is still a certain amount of scatter in the experimental results for  $I$ . This is not serious, however, because  $I$  enters the formulas for the stopping power and other relevant parameters only logarithmically.

In table 5 we list a set of ionization potentials that were made the basis for the computation of other parameters. Unavoidably, the selection had to be somewhat arbitrary, but it was done with care so as to be representative of the experimental situation. Main reliance was placed on results obtained with high-energy protons (Mather and Segrè's result for aluminum was omitted and Thompson's results for graphite was accepted). Inasmuch as the application of this report is principally to cavity ionization chambers for electrons, it seemed reasonable to choose  $I$  values obtained with protons that have velocities matching as nearly as possible the velocities of the electrons of interest. Moreover, at high energies the interpretation of the experiments is simplified in that smaller shell corrections are needed.

The selection of table 5 was completed before the recent results of Zrelov and Stoletov, and of Barkas and von Friesscn became available. It has turned out, however, that these new data are in very good agreement with our selection, so that there was no need for a revision. It should be kept in mind, however, that further theoretical and experimental work are needed before arriving at definite  $I$ -values.

TABLE 5. Selected  $I$  values <sup>a</sup>

Element	$I$ ev	Author <sup>b</sup>	Notes
1 H.....	20.7	T.....	Liquid
3 Li.....	38	BS.....	
4 Be.....	67	BS.....	
6 C.....	78.4	T.....	Graphite
7 N.....	85.1	T.....	Liquid
Air.....	85	Text.....	Gas
8 O.....	98.3	T.....	Liquid
13 Al.....	164	Text.....	
26 Fe.....	264	BS.....	
29 Cu.....	306	BS, MS.....	
47 Ag.....	462	BS.....	
50 Sn.....	517	BS.....	
74 W.....	750	BS.....	
82 Pb.....	812	BS, MS.....	
92 U.....	945	BS.....	

<sup>a</sup> All of table 3 is to be considered part of this table.

<sup>b</sup> BS=Bakker and Segrè (1951) renormalized, T=Thompson (1952) renormalized, MS=Mather and Segrè (1951).

## 4. Theoretical Values of $_m s$

Section 2 presents us with a theory for cavity chambers. Section 3 gives us the data necessary to evaluate the constant  $s$  that appears in the theory. We will now examine how to calculate  $s$  in order to be able to compare it with experimental data in section 5.

It is convenient to deal with  $_m s = s\rho_a/\rho_Z$  rather than  $s$ , because the stopping powers contain the density as a factor. Removal of this factor gives numbers having the same order of magnitude for all phases. The Bragg-Gray formula can be changed to incorporate  $_m s$  by using energies absorbed or ionization produced per unit mass of material,  $_m E$  or  $_m J$ .

$$_m E_a = \frac{1}{_m s} _m E_z \quad (29)$$

$$_m E_z = _m s w _m J \quad (30)$$

The notation  $_m s_g^w$  will be used when it is necessary to specify the nature of the wall material,  $w$ , and the gas,  $g$ .

### 4.1. Basic Bragg-Gray Principle

Laurence's result for the average of the stopping powers that is required for  $_m s$  is given by equation (11) due to Spencer and Attix. The equation gives  $f=1/_m s$  rather than  $_m s$ . This is convenient because the observed quantity,  $_m J$ , is proportional to  $1/_m s$ ; see (29). When averages are taken, they have to be of  $1/_m s$ . Using equation (20) for the stop-

ping power and using the stopping number per electron  $eB$  defined in footnote 13, we can rewrite (11) as

$$f_z(T_0) = \frac{(Z/A)_a}{(Z/A)_z} \left[ 1 + \frac{1}{T_0} \int_0^{T_0} \ln \frac{I_z + \frac{\delta_z}{2}}{eB_z} dT \right] \quad (31)$$

$\delta_a$  does not appear in this equation because the polarization effect in the gas in the chamber is negligible except at very high energies.

It is convenient to introduce two functions  $b_z(T_0)$  and  $d_z(T_0)$  defined so that

$$f_z(T_0) = \frac{(Z/A)_a}{(Z/A)_z} \left[ 1 + b_z(T_0) \ln \frac{I_z}{I_a} + d_z(T_0) \right]. \quad (32)$$

Both functions have been calculated for useful energies and materials.  $b_z$  is given in table 6;  $d_z$  is given in table 7. Calculations of  $eB$  by Nelms (1956) show that the low energy limit of  $eB$ ,  $\ln(\frac{1}{2}e)^{1/2}(T/I)$ , is accurate to within 4 percent up to 3 Mev. The low energy limit can be integrated to give

$$b_z(T_0) \approx \frac{Ei \left( \ln \sqrt{\frac{e}{2}} \frac{T_0}{I_z} \right)}{\sqrt{\frac{e}{2}} \frac{T_0}{I_z}}, \quad (33)$$

$Ei$  is the exponential integral<sup>14</sup> that has been tabulated; e.g., by the Federal Works Agency (1940).

Although  $b_z$  depends on the  $I$  values chosen for the calculation, the dependence is only logarithmic. The greatest uncertainty in experimental  $I$  values is only about 25 percent. The resulting uncertainty in  $b_z$  is only a few percent. This is satisfactory because the resulting uncertainties in  $f$  are smaller still. The important dependence of the second term of (32) on the  $I$  values is through the factor  $\ln I_z/I_a$ . This factor is left to be evaluated by the experimenter so he can use the latest knowledge about the  $I$  values.  $I$  values based on Bakker and Segrè's work were used to compute  $b_z$ .

<sup>14</sup>  $Ei(x) = \int_{-\infty}^x \frac{e^{-u} du}{u}$ ;  $Ei(-x) = -\int_x^{\infty} \frac{e^{-u} du}{u}$ .

TABLE 6.  $b_z(T_0)$

$T_0$ Mev	C	Al	Cu	Sn	Pb
0.100	0.16430	0.18772	0.22408	0.28880	0.30712
.200	.14764	.16000	.19262	.23148	.24916
.300	.13938	.15344	.17812	.20786	.22408
.327	.13780	.15346	.17540	.20320	.21936
.400	.13404	.14862	.16908	.19388	.20896
.500	.13020	.14366	.16264	.18424	.19836
.600	.12722	.13980	.15774	.17702	.19048
.654	.12598	.13832	.15570	.17380	.18706
.700	.12480	.13664	.15380	.17132	.18424
.800	.12278	.13396	.15054	.16666	.17912
1.000	.11956	.12968	.14536	.15940	.17104
1.200	.11704	.12634	.14136	.15392	.16490
1.308	.11580	.12478	.13940	.15120	.16204
1.500	.11410	.12246	-----	.14764	.15790

TABLE 7.  $d_z(T_0)$

$T_0$ Mev	C	Al
0.300	0.00115	0.00006
.327	.00154	.00013
.400	.00272	.00051
.500	.00439	.00120
.600	.00606	.00203
.654	.00696	.00253
.700	.00770	.00291
.800	.00930	.00397
1.000	.01240	.00597
1.100	.01391	.00693
1.200	.01537	.00790
1.308	.01692	.00891

$d_z$  depends quite strongly on the  $I$  values chosen for the calculation. The dependence is a complicated one so that a separation into two factors as in the other term is not possible. Sternheimer calculated two sets of the polarization correction,  $\delta$  (1952, 1956). One was for the original Bakker-Segrè  $I$  values, the other was for  $I$  values from Sachs and Richardson. Since the first of these have been re-normalized and the latter rejected as being in error, fresh calculations are generally needed. Fortunately, however, the  $d_z$  term is relatively small in the cases we wish to consider except for chamber walls of carbon and aluminum. The  $I$  values for these materials used in the 1956 calculations were 78 and 163 ev. This is excellent agreement with Thompson's  $I_C=78.4$  ev (table 2) and our adopted value of  $I_{Al}=164$  ev. These calculations were used to obtain the  $d_z$  in table 7.  $d_z$  for other materials will be neglected. The worst error will be for copper and will be only a few tenths of a percent in  $f$ .

Equation (11) and its descendents (31) and (32) apply to an emitter of monoenergetic electrons of energy  $T_0$  distributed uniformly through the chamber walls. It can be used, e.g., for a chamber exposed to gamma rays if the chief interaction in the chamber walls is photoelectric absorption because the photoelectrons will all have closely the same energy. For other cases,  $f_z(T_0)$  must be averaged over the spectrum of initial energies as shown in equation (12). This was done for the spectrum of recoil electrons from Compton scattering. The results can be expressed by

$$\bar{f}_z(T\gamma) = \frac{(Z/A)_a}{(Z/A)_z} [1 + a_z(T\gamma) \ln I_z/I_a + D_z(T\gamma)], \quad (34)$$

where  $a_z$  and  $D_z$  are averages over  $b_z$  and  $d_z$ , respectively. The same remarks apply to  $a_z$  and  $D_z$  as applied to  $b_z$  and  $d_z$ . Results for  $a_z$  are presented in table 8, for  $D_z$  in table 9.

TABLE 8.  $a_z(T\gamma)$

$T\gamma$ Mev	C	Al	Cu	Sn	Pb
0.15	0.20020	0.23796	0.30636	0.46582	0.48180
.25	.17318	.20012	.24436	.33174	.34982
.4	.15450	.17552	.20950	.25848	.27864
.6	.14210	.15904	.18360	.21852	.23478
1.0	.12948	.14272	.16184	.18396	.19780
1.5	.12166	.13240	.14894	.16496	.17702
2.0	.11672	.12592			
2.5	.11334	.12150			

TABLE 9.  $D_z(T\gamma)$

$T\gamma$ Mev	C	Al
0.4	0.00014	0.0000
.6	.00148	.00022
1.0	.00597	.00219
1.5	.01167	.00567

## 4.2. Modified Theory of Spencer and Attix

The Spencer-Attix result for the inverse cavity chamber stopping power ratio is given in equation (15). It can be treated in the same way we have just treated (11) (see footnote 11). The result may be written in a form very much like (32).

$$f_z(T_0, \Delta) = \frac{(Z/A)_a}{(Z/A)_z} [1 + c_z(T_0, \Delta) \ln \frac{I_z}{I_a} + d_z(T_0)]. \quad (35)$$

The dependence on  $\Delta$ , the energy of the electron that can just cross the average dimension of the chamber, is through the factor  $c_z(T_0, \Delta)$  that replaces  $b_z(T_0)$ . Like  $b_z$ ,  $c_z$  is not a sensitive function of the  $I$  values.  $I$  values based on the work of Bakker and Segrè were used to calculate  $c_z$ . The results are in table 10.

The term in (35) that represents the polarization effect is the same as in (32). The reason that it is the same is that the changes in the flux of cavity-traversing electrons that Spencer and Attix allowed for are of importance for low energy electrons for which the polarization effect is negligible.

$f_z(T_0, \Delta)$  applies to a monoenergetic emitter so an averaging process is necessary whenever there is a spectrum of starting energies. Averaging is difficult because there is so little data for  $f_z(T_0, \Delta)$ . The process that has been used is based on the fact that  $f_z(T_0, \Delta)/f_z(T_0)$  is found to be relatively insensitive to energy. If it is assumed to be constant at the value it has for  $\bar{T}$ , the average energy of the electrons in the starting spectrum, then the average value is given by

$$\bar{f}_z(T\gamma, \Delta) = \frac{f_z(\bar{T}, \Delta)}{f_z(\bar{T})} \bar{f}_z(T\gamma). \quad (36)$$

and  $\bar{f}_z(T\gamma)$  can be obtained from (34). For Compton recoil electrons  $\bar{T} = h\nu(\sigma_a/\sigma)$  where  $\sigma_a$  and  $\sigma$  are the Compton absorption and total coefficients, respectively.

TABLE 10.  $c_z(T_0, \Delta)$

$T_0$ Mev	$\Delta$ Kev	C	Al	Cu	Sn	Pb
1.308	81.8	0.12360	0.13372	0.14958	0.16032	0.17416
	40.9	.12956	.14110	.15895	.17168	.18812
	20.4	.13716	.15058	.17130	.18668	.20682
	10.2	.14594	.16192	.18630	.20614	.23290
	5.1	.15705	.17692	.20764	.23402	.27172
	2.56	.17046	.19468	.23744	.27664	
	.654	81.8	.12806	.14014	.15674	.16930
40.9		.13510	.14876	.16782	.18246	.20124
20.4		.14242	.15810	.18018	.19776	.22092
10.2		.15200	.17050	.19888	.21896	.24838
5.1		.16332	.18576	.21910	.24852	.28914
2.56		.17772	.20370	.25118	.29454	
.327		81.8	.13282	.14672	.16460	.17830
	40.9	.14006	.15560	.17596	.19262	.21284
	20.4	.14874	.16660	.19040	.21028	.23510
	10.2	.15796	.17880	.20738	.23196	.26382
	5.1	.17018	.19526	.23128	.26390	.30536
	2.56	.18530	.21690	.26512	.31244	

From (36) it is evident that  $f_z(\bar{T}, \Delta)/f_z(\bar{T})$  is a measure of how much the modified theory of Spencer and Attix differs from the original theory of Gray and of Laurence. The ratio is plotted in figures 3.a-e. For materials close to air in atomic number, there is only a few tenths percent difference. For a material as different as lead, there is a difference of as much as 20 percent.

Table 11 gives the range  $R$  of an electron of energy  $\Delta$ . In the Spencer-Attix theory,  $\Delta$  is fixed by requiring  $R$  to equal the average diameter of the chamber.

TABLE 11. Range of low energy electrons

$\Delta$	$R$
Kev	cm STP air
2.56	0.015
5.1	.051
10.2	.19
20.4	.64
40.9	2.2
81.8	7.3

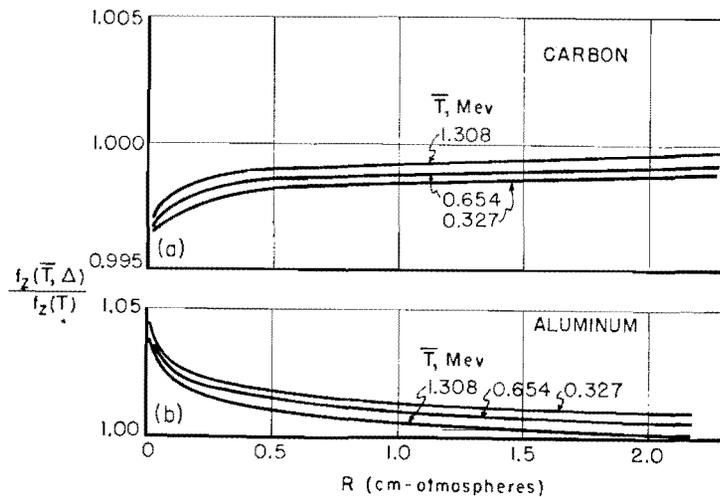


FIGURE 3.a and 3.b. Predictions of Spencer-Attix theory for variation of  $mJ$  with pressure.

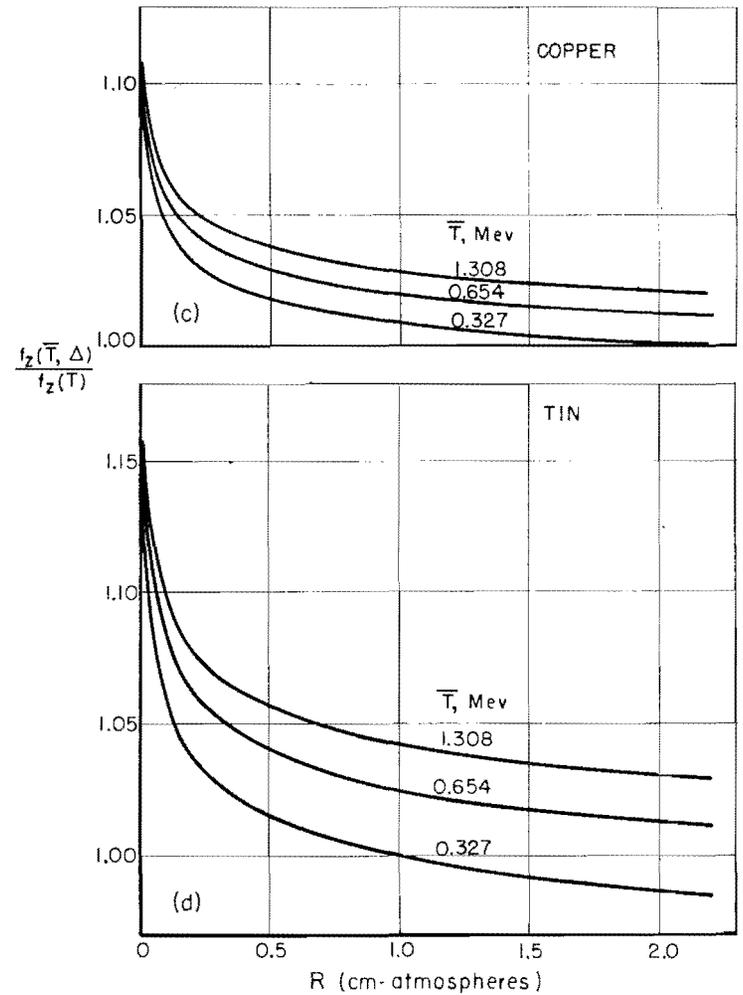


FIGURE 3.c and 3.d. Predictions of Spencer-Attix theory for variation of  $mJ$  with pressure.

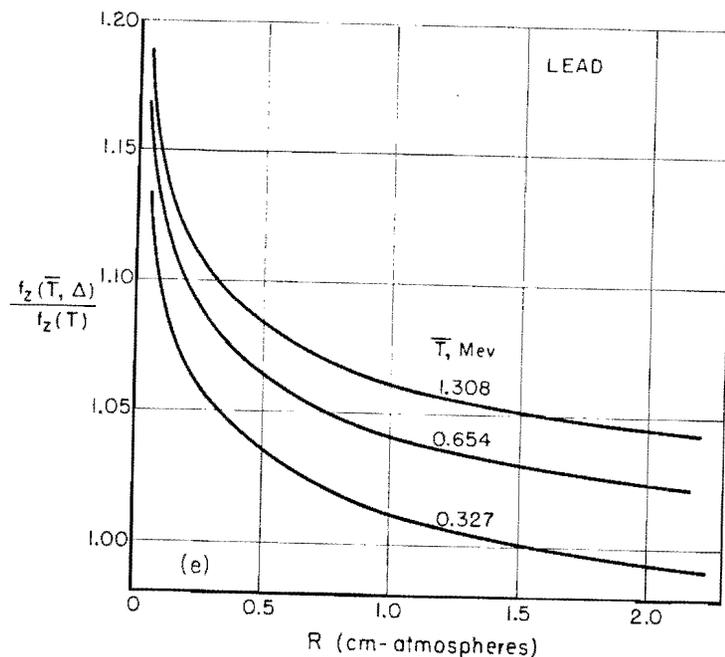


FIGURE 3.e. Predictions of Spencer-Attix theory for variation of  ${}_mJ$  with pressure.

#### 4.3. Variation of ${}_mJ$ With Pressure

Since  ${}_mJ$  is proportional to  $f=1/{}_m s$ , equations (35) and (36) of the Spencer-Attix theory predict that  ${}_mJ$  will vary with  $\Delta$ .  $\Delta$  is the energy of an electron whose range equals the average diameter of the chamber. It, and hence  ${}_mJ$ , can be varied by changing either the chamber size or the pressure of the gas in the chamber. No such variation is predicted by the Gray or Laurence theory. In fact, they require the absence of such a variation as prerequisite for their proper application.

From equation (36) we see that  ${}_mJ$  will be proportional to  $f_z(\bar{T}, \Delta)/f_z(\bar{T})$  which can be written

$$\frac{f_z(\bar{T}, \Delta)}{f_z(\bar{T})} = 1 + \left[ \frac{c_2(\bar{T}, \Delta) - b_2(\bar{T})}{f_z(\bar{T})} \right] \ln \frac{I_z}{I_a} \quad (37)$$

This ratio is plotted in figures 3.a-e versus the electron range  $R$  in air that corresponds to the energy  $\Delta$ . For application

to experimental data  $R$  can be taken to be the product of the mean diameter of the chamber (in centimeters) and the air pressure in the cavity in atmospheres.

## 5. Cavity Chamber Measurements

In this section we will compare experimental results for  ${}_m s$  with values calculated by the methods of the preceding section. There are several types of experiments that give useful information. We will first describe the data concerning the variation of  ${}_mJ$  with pressure since there is an important difference between the newer and older theories on this point. Then the relative values of  ${}_m s$  that can be obtained from comparing chambers that are identical except as to wall material or from comparing different gases in a single chamber will be studied. Finally, the absolute values of  ${}_m s$  that can be obtained by separately measuring each of the other factors in the Bragg-Gray equation will be considered.

### 5.1. Variation of ${}_mJ$ With Pressure

A distinctive feature of the modified theories of Spencer and Attix and of Burch as distinguished from the theories of Gray and of Laurence is that the former predict that  ${}_mJ$  will vary with pressure (for constant cavity size, or with cavity size at constant pressure) even for very low pressures (or cavity size). We will now review the experimental evidence on this point. The existence and magnitude of the effect is important for verifying the theory, for interpreting experimentally measured  ${}_m s$ 's, and for supplying information needed in the interpretation of measurements with the Failla (1937) extrapolation chamber.

Gray (1936) stated: "The (Bragg-Gray) equation may be considered valid in any circumstances in which the ionization remains proportional to the pressure as the pressure is reduced below the normal value." In other words, for sufficiently small pressures  ${}_mJ$  should achieve a constant value, independent of further reduction in pressure (or cavity size at constant pressure). Gray demonstrated experimentally that this was so, within 1 percent, for air-filled graphite chambers of 0.1 and 2.0 cm<sup>3</sup> volume, with gamma rays from radon. Reduction of the pressure from 74 cm to 10 cm Hg produced no significant variation in  ${}_mJ$ . On the other hand, a similar measurement in a lead-walled cavity of 0.1 cm<sup>3</sup> revealed a 7-percent increase in  ${}_mJ$  for the same pressure

decrease. Gray concluded that the (basic) Bragg-Gray equation was not valid for this situation, but no attempt was made to explain the effect.

One interesting feature of Gray's result for lead was that the plot of  $mJ$  versus pressure did not tend to level off toward a constant value at low pressures. If anything, it appeared to be slightly concave upward but was roughly a straight line with negative slope. From these data, there is no promise that constancy can be achieved by going to still lower pressures. Others have obtained similar results.

Ibrahim and Wilson (1952) used a flat extrapolation chamber, varying the gap width down to about 0.5 mm at 1 atmosphere air pressure. X-rays of moderate filtration, up to 124 keV (effective), were employed to irradiate the chamber. The walls were of graphite, aluminum, copper, and some molded compositions. Graphite showed linear behavior, as in Gray's results, while the other materials, having  $Z$  greater than that of air, again gave an increasing  $mJ$  with decreasing gap. This experiment was complicated by the fact that the X-rays were of low energies, so that the average starting energy of the primary electrons was not large compared to the cavity size, as required for proper application of cavity theory. Thus much of the apparent rise in  $mJ$  with decreasing gap size is caused by the transition from the predominance of electrons originating in the air gap to that of electrons originating in the wall. There are more of the latter, due to the photoelectric effect, hence the exaggerated rise in  $mJ$ . A further complication, also augmenting the rise in  $mJ$ , is the loss of electrons out the edge of the chamber, which had an electric-field guard ring of lucite rather than of the wall materials under consideration. These losses become progressively greater as the gap width is increased.

Attix, DeLaVergne, and Ritz (1958) carried out a similar experiment with a flat extrapolation chamber of an improved design, having guard rings of the same material as the rest of the walls, and less extraneous material in the radiation beam to produce scattered photons. Walls of carbon, aluminum, copper, tin, and lead were studied with heavily filtered X-ray energies from 38 to 206 keV (effective) and with gamma rays of 411 (Au<sup>198</sup>), 670 (Cs<sup>137</sup>), and 1,250 keV (Co<sup>60</sup>).

The experimental results were generally similar to those of Ibrahim and Wilson at the lower energies. For walls other than graphite, the steep rise of  $mJ$  was observed as the gap width was reduced from 10 mm to 0.5 mm. Some edge losses of electrons were observed in spite of the improved chamber design. For graphite  $mJ$  was found to rise slightly

with reduction in gap width for X-ray energies greater than 100 keV because of these edge losses; however, at 38 keV a very marked decrease in  $mJ$  was observed for graphite, contrary to all the other results.  $mJ$  decreased by about 24 percent as the gap was reduced from 1.0 cm to 0.05 cm. This can be ascribed to the previously mentioned influence of electrons originating in the air. Their effect was particularly pronounced in this case because the air has considerably more photoelectric effect than the wall and the resulting photoelectrons have much longer ranges than the Compton recoils. It is interesting to notice that Ibrahim and Wilson observed no similar effect with their 38 keV X-ray measurements with graphite. The difference may be due to their greater spectral width, as they used lightly filtered 100 Kvp X-rays while the present work employed 50 Kvp X-rays, heavily filtered.

The gamma ray measurements were found to give more modest rises of  $mJ$  with decreasing gap width, the maximum being about 50 percent for lead walls. An auxiliary experiment was tried with the Co<sup>60</sup> gamma ray measurements to determine the size of the electron edge losses. Rings of the same materials as the walls were fitted into the chamber to close the edges of the gap. The results showed that most of the rise in  $mJ$  with gamma rays was due to this cause so that only about 9 percent remained for lead. The modified theory of Spencer and Attix (with  $I$  values based on the original Bakker-Segrè work) was then compared and found to predict the pressure variation of  $mJ$  fairly well for all materials. There was no observable tendency for the slope of  $mJ$  to "flatten out" at small gap widths except for graphite which appeared flat for all widths after the edge losses were eliminated. The experimental results for Co<sup>60</sup> with the chambers with side walls are shown in figure 4. The curves for the Spencer-Attix theory were calculated as in section 4.2. with the values of  $I$  from table 5.

Cormack and Johns (1954) also employed a flat extrapolation chamber with walls of carbon, aluminum, copper, tin, and lead using a Co<sup>60</sup> gamma ray source and 22 Mv X-rays. They avoided edge losses by means of spacing rings. Their results indicate complete proportionality between ionization and separation for all materials. This result does not agree with any of the other measurements given in the literature for high atomic number walls; however, the reason for the discrepancy is not apparent.

Larson (1956) used a flat chamber with fixed plate separation of about 1.5 cm and walls of aluminum and copper. Air pressure was varied from one atmosphere down to 2 mm

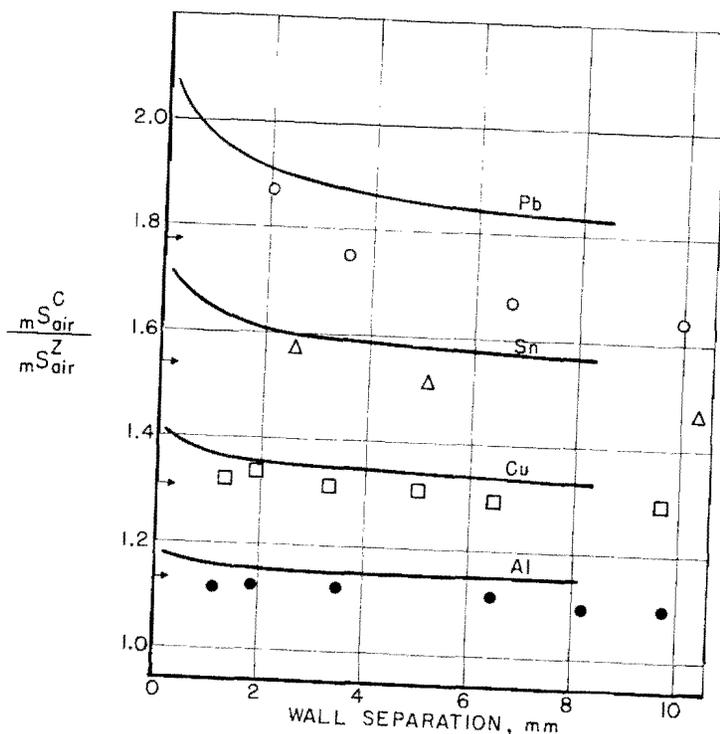


FIGURE 4. Variation of  $mJ$  with cavity size.

$mJ$  is proportional to  $1/r_s$ . The points are the experimental data of Attix, DeLaverne, and Ritz (1958) for  $Co^{60}$  and extrapolation chambers whose sides were enclosed with the same material as the plates. The solid lines are the theory of section 4.2.

Hg going to cavity "sizes" of about one-tenth of the minimum usable cavity size attained in previous studies. This is apparently about the minimum usable cavity size for reliable results, because at lower pressures Larson found that he could not attain current saturation even with the optimum flat chamber design.<sup>15</sup>

Larson found that, although  $mJ$  did not become constant at low pressures, it increased only slowly and with a constant slope. Thus it was possible to extrapolate his results to zero

<sup>15</sup> Sievert (1940), Taylor (1951), and Wilson (1954) also encountered this difficulty for chambers of this order of pressure-size. Presumably some of their difficulties may have been due to multiplication taking place in the gas. The situation is further complicated by the presence of the Greening effect (1954) in which the observed current is not entirely from ionization but is partially a current of very low energy electrons emigrating from one electrode to the other under the influence of the electric field. These electrons are the low energy tail on the spectrum of charged particles existing in the chamber walls due to the interactions with the photons. Measurements of the spectra of these electrons have been reported by Nelson et al. (1959) and by Finston et al. (1959).

pressure with some degree of confidence. Larson used monoenergetic characteristic X-rays as his source; the energies were between 8 and 34 kev. It would be of interest to do a similar experiment where the pressure is reduced to these low values for gamma rays where the starting energies of the primary electrons are much greater than the chamber dimensions. Thus one would be effectively investigating even smaller cavities than those achieved by Larson relative to the electron ranges present. A region of constant  $mJ$  might be observable under these conditions, if indeed it exists at all.

Greening (1957) studied the pressure variation of  $mJ$  in small cylindrical chambers about 1 cm by 1 cm in size. The walls were of carbon, copper, tin, tantalum, and lead; the sources were  $Co^{60}$ ,  $Cs^{137}$ , and  $Au^{198}$ . As the pressure was reduced to about 5 cm Hg, Greening observed a variation in  $mJ$  for copper, tin, and lead that compared well with the theoretical predictions of Spencer and Attix (1955) and had an increasingly steep negative slope with decreasing pressure.

Whyte (1957) used a chamber 7 cm long by 5 cm diameter, with walls of beryllium, carbon, aluminum, and copper; the source was  $Co^{60}$ . The pressure was varied from one atmosphere down to about 8 cm Hg. Over this range  $mJ$  for copper and aluminum was observed to increase with nearly constant slope with only a trace of the upward concavity observed by Greening and by Attix, DeLaverne, and Ritz. For both carbon and beryllium a corresponding decrease in  $mJ$  was observed. For carbon it was only a few tenths of a percent which could easily have been missed in less accurate experiments previously described.

Attix and Ritz (1957) made some pressure variation measurements in the course of a determination of the radium gamma ray emission. Their chamber was cylindrical, 4 cm diameter by 5 cm long, with walls of carbon, aluminum, and copper. The source was  $Co^{60}$ . The pressure was varied from one atmosphere down to 0.1 atmosphere. The graphite chamber ionization  $mJ$  was observed to decrease by 0.15 percent which was somewhat less of a change than that observed by Whyte, but nevertheless significant.  $mJ$  for aluminum and copper walls was found to increase with a steepening slope as the pressure was reduced. The slope changed more rapidly than predicted by theory for pressures above about 0.3 atmosphere. This discrepancy probably results from the large size of the cavity; at the higher pressure the cavity size restriction on the theory is not adequately fulfilled.

The results on the variation of  $mJ$  with pressure or cavity size can be summarized as follows: For energies in the X-ray

range,  ${}_mJ$  experimentally varies linearly with pressure-size for small pressure-size. With the exception of the work of Cormack and Johns, experiments for energies in the gamma ray range show that  ${}_mJ$  as a function of pressure-size has an increasing negative slope as the pressure-size is reduced.

The nature of the agreement between experiment and the Spencer-Attix theory can be seen in figures 4 and 5. The points in figure 4 are the experimental data of Attix, DeLaVergne, and Ritz with  $\text{Co}^{60}$  for chambers completely enclosed with the wall material being studied. The solid lines are the predictions of Spencer and Attix, for the new  $I$  values of table 5 as discussed in section 4.2. For these curves,  $\Delta$  was taken as the energy of an electron whose range was twice the wall separation. (The authors used a range equal to the wall separation; the present choice was felt to give a better average value of  $\Delta$  for the cavity.) Evidently the experimental points are tending toward the theoretical curve at small wall separations.

The difference between the theory and experiment can be analyzed as suggested in section 2.6.c. In figure 5 are plotted the experimental values of  ${}_mS_{\text{air}}^C / {}_mS_{\text{air}}^Z$  minus the difference between the Spencer-Attix theory and the basic Bragg-Gray value for the same quantity. The curvature of the  ${}_mJ$  curves is removed by this procedure and a linear extrapolation can be made to zero wall separation. Indeed, as shown, this linear extrapolation passes through the basic Bragg-Gray value at zero wall separation. This procedure is in the spirit suggested in the introduction (section 1). The Laurence-Bragg-Gray theory is an approximation to cavity chamber theory. The Spencer-Attix theory is a further approximation that takes into account delta ray effects, but does not account for perturbations in the cavity traversing flux due to the presence of the cavity. When the Spencer-Attix theory is treated as a correction, as above, the remaining difference from the Bragg-Gray-Laurence value is interpreted as due to the flux perturbation and will presumably be explained by a better approximation that has not yet been made.

## 5.2. Chambers With Different Atomic-Number Walls

When chambers with walls made of different atomic number materials are exposed to identical fluxes of radiation, the absorbed doses in the walls are proportional to the mass energy-absorption coefficients of the wall materials. If the chambers are filled with the same gas,  $w$  can be assumed to be the same for all of them. Then

$$\frac{{}_mS_a^Z}{{}_mS_a^{Z'}} = \frac{{}_m\mu_{en}^Z}{{}_m\mu_{en}^{Z'}} \frac{J_{Z'}}{J_Z}, \quad (38)$$

where  ${}_m\mu_{en}$  is the mass energy-absorption coefficient. Since in each exposure the results are given at constant air density, the ratio of the  $J$ 's can be used in place of the ratio of the  ${}_mJ$ 's.

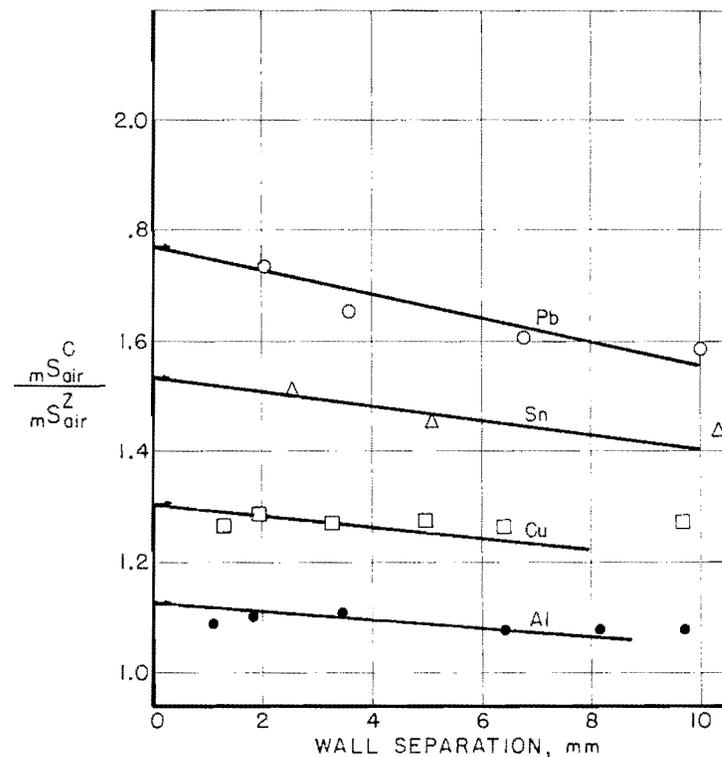


FIGURE 5. Effect of flux perturbation on  ${}_mJ$ .

The points are the experimental data of figure 4 minus the difference between the Spencer-Attix theory and the Bragg-Gray-Laurence theory.

### a. Energy-Absorption Coefficients

${}_m\mu_{en}$  is obtained from experimental data on gamma ray absorption coefficients. There is, however, lack of agreement between different authors as to how this is to be done. In the theory of absorption measurements (see, e.g., Johns

and Laughlin, 1956) there appears a quantity called the true absorption coefficient and defined by

$$m\mu_{en} = m\tau + m\sigma_a + m\kappa \left(1 - \frac{2m_0c^2}{h\nu}\right), \quad (39)$$

where  $m\tau$ ,  $m\sigma_a$ , and  $m\kappa$  are the mass absorption coefficients for photoelectric absorption, Compton absorption, and pair production by photons of energy  $h\nu$ . Many authors identify the energy-absorption coefficient with the true absorption coefficient (see, e.g., Spiers, 1956). Marinelli (1953), however, said the photoelectric component, for the  $K$  shell, should be given by

$$m\tau^{(K)} \left(1 - \frac{fh\nu_K}{h\nu}\right), \quad (40)$$

where  $m\tau^{(K)}$  is the mass absorption coefficient for photoelectric absorption (in the  $K$  shell) for photons of energy  $h\nu$ ,  $f$  is the  $K$  shell fluorescent yield, and  $h\nu_K$  is the  $K$  shell binding energy. This assumes that the chamber walls are thin enough that most of the fluorescent radiation emitted in filling the  $K$  shell vacancy escapes from the chamber. The other authors implicitly assume that the wall of the chamber is thick enough to prevent escape of the fluorescent radiation. The actual case must lie between these two extremes. The fluorescent X-rays may have an absorption coefficient greater, equal to, or less than that of the primary rays so that little of it, some of it, or most of it (respectively) may escape from the chamber.

In order to get some idea of the effect of the escape or nonescape of fluorescent radiation, we will consider a simple case in which the effect can be computed. The case chosen is that of the dose rate at the center of an infinite slab of thickness  $2t$  in the direction of a beam of gamma rays. Physically this would apply to the center of a chamber whose walls were of thickness  $t$  and whose dimensions perpendicular to the beam were very large. The latter is not usually true so the result cannot often be applied directly, but it does give one a deeper understanding of the problem. A convenient way of presenting the result of this calculation is to give a factor  $g$  that can be inserted in (40) to indicate the extent of the escape of fluorescent rays,

$$m\tau^{(K)} \left(1 - g \frac{fh\nu_K}{h\nu}\right). \quad (40.a)$$

This method of presentation is convenient because it turns out that  $g$  is a function of just the thickness  $t$  and the ratio of the absorption coefficients of the fluorescent and primary rays. From (40) and (40.a) we find that the ratio of the dose due to the fluorescent rays to that due to the primary rays is

$$(1-g) \frac{fh\nu_K}{h\nu} \frac{1}{1-fh\nu_K/h\nu}. \quad (40.b)$$

The maximum value of  $g$  will be one and occurs when all the fluorescent radiation escapes. Capture of fluorescent radiation leads to values of  $g$  that are less than one and may even be negative.

When  $h\nu$  is small enough that the bracket in (40) is significantly different from unity, photoelectric absorption far exceeds any other type of gamma ray interaction. The absorption of the gamma rays can be represented as exponential and no allowance need be made for a buildup of scattered radiation. The fluorescent X-rays will be emitted isotropically. The energy of the X-rays will be approximately  $h\nu_K$ ; actually it will be slightly less. Under these conditions, in the middle of a block of material of thickness  $2t$  exposed to a beam of gamma rays<sup>16</sup>

$$g = 1 - \frac{\tau_K}{2\tau} \left\{ \ln \left| \frac{\tau + \tau_K}{\tau - \tau_K} \right| + Ei(\tau t - \tau_K t) - Ei(-\tau t - \tau_K t) - (e^{\tau t} - e^{-\tau t}) Ei(-\tau_K t) \right\} \quad (41)$$

where  $\tau_K$  is the absorption (practically all by photoelectric effect) coefficient of the fluorescent X-rays. If  $\tau t \ll 1$ , then

$$g \approx e^{-\tau_K t} + \tau_K t Ei(-\tau_K t) \quad (41.a)$$

Figure 6 shows the behavior of  $g$  as a function of  $\tau t$  for different relative values of  $\tau_K$  and  $\tau$ . For very small  $t$  (relative to  $1/\tau_K$  or  $1/\tau$ ),  $g$  is slightly less than 1, meaning

<sup>16</sup> If  $x$  is distance measured into the slab and  $y$  is distance measured perpendicular to  $x$  then a unit flux of primary gamma rays will produce a fluorescence radiation dose equal to

$$\int_0^{2t} \int_0^\infty e^{-\tau z} f r 2\pi y dy dz \cdot \frac{\tau_K h\nu_K e^{-\tau_K \sqrt{(t-x)^2 + y^2}}}{4\pi[(t-x)^2 + y^2]}.$$

Equation (41) can be obtained from this and equations (40) and (40.b). Some details of the calculation are given by Goldstein (1954) who treated the problem for  $t$  infinite.  $Ei$  is the notation for the exponential integral; see footnote 14.

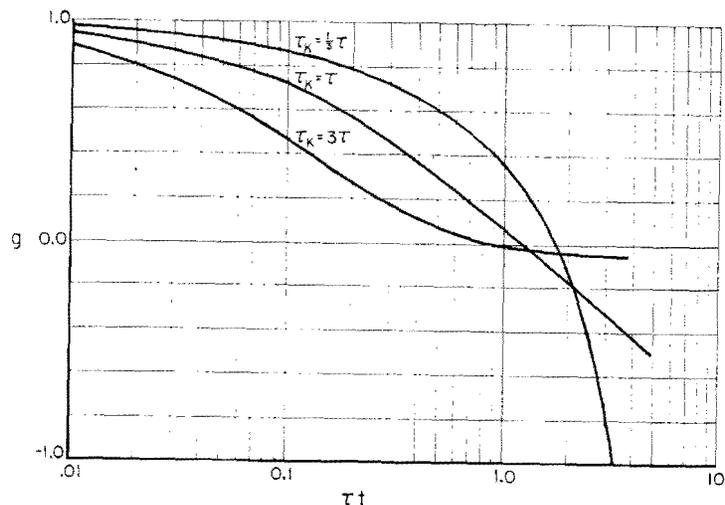


FIGURE 6. The correction for escape of fluorescence radiation.

that most of the fluorescent X-rays escape. For larger  $t$ , if  $\tau_k > \tau$ ,  $g$  becomes quite small; it actually becomes *slightly* negative near  $\tau t = 1$ . This means that most of the X-rays are captured near their point of origin. The negative value indicates that the X-ray dose at a point is due principally to those X-rays released before that point in the beam. If  $\tau_k < \tau$ ,  $g$  becomes negative without limit as  $t$  increases. This means that X-rays reach a given point from throughout the medium and keep on increasing with the size of the medium.

Effects in the  $L$  shell can be treated as follows: In the bracket of (40.a) there should be a third term, similar to the first but to allow for the production of  $L$  X-rays following photoelectric absorption in the  $K$  shell. In all cases of practical importance up to the present, however,  $\tau_L$  is so large that the  $g$  factor can be considered zero so the term vanishes. In  $m\mu_{en}$  there should also be a term of the form of (40.a) but with  $m\tau^{(L)}$  for the  $L$  shell; since  $g$  can be considered zero, the bracket will equal unity.

The question of escape or non-escape of secondary radiation must also be considered for Compton scattering and pair production. The Compton component of  $m\mu_{en}$  is usually given as  $m\sigma_a = m\sigma - m\sigma_s$  where the symbols are the macroscopic Compton energy-absorption, total, and scattering cross sections per unit mass, respectively (Johns and Laughlin, 1956).

This assumes complete escape of the scattered rays. The effect of secondary absorption can be given in a way similar to that used in (40.a) by saying that a factor  $j$  in  $m\sigma - j m\sigma_s$  measures the number of rays that escape; i.e., the dose due to scattered rays is proportional to  $m\sigma_s (1-j)$ .  $j=1$  means that all the scattered rays escaped. Calculation of  $j$  is very difficult. The scattered rays are not monoenergetic nor are they emitted isotropically. To make a rough estimate of  $j$ , let us suppose that the scattered rays continue to move in the same direction as the original rays, that they all have the energy  $(m\sigma_s/m\sigma)(h\nu)$  equal to the average energy of the scattered rays, and that  $\sigma$  is much larger than  $\tau + \kappa$ . Then

$$j = 1 - \sigma_a' t, \quad (42)$$

where the prime refers to the scattered rays. The wall thickness of a chamber will be about equal to the maximum range of the secondary electrons; this will make  $\sigma t$  about 0.03. The average energy of the scattered rays is about half that of the primary rays and their absorption coefficient about 50 percent greater. Roughly  $\sigma_a \approx \sigma_s \approx \frac{1}{2}\sigma$ . This gives  $j = 0.98$ . Although this is just a crude estimate, it indicates that care is needed in interpreting experiments that make use of  $m\mu_{en}$ . It should be noted that equation (38) requires only a ratio of energy absorption coefficients. Where Compton scattering is the predominant interaction, the correction for secondary ray absorption will amount to very nearly equal factors in both coefficients and cancel out of the ratio.

In the case of pair production, (39) assumes that the annihilation radiation escapes. A correction for its capture could be introduced. The annihilation radiation would be monoenergetic and most of it would be isotropic, but calculation of the amount escaping would be difficult because the rays would be emitted at the end of the positron track rather than at the point where the gamma ray was absorbed. Fortunately, in most cases of current interest,  $\kappa$  is small compared with  $\tau + \sigma$  and we can neglect the capture without introducing appreciable error.

When an experimenter corrects for absorption by adding material of the same composition as the chamber to the outside of the chamber and then extrapolating back to zero absorber, he is really correcting for two things. He corrects for the absorption of the primary radiation and for the capture of the different secondary radiations. By extrapolating to zero absorber thickness, he converts his measurements to the conditions where most of the secondary rays (except,

e.g.,  $L$  X-rays) escape. Under these conditions  $m\mu_{en}$  is properly given by

$$m\mu_{en} = m\tau \left( 1 - \frac{fh\nu_{\kappa} m\tau^{(K)}}{h\nu m\tau_a} \right) + m\sigma_a + m\kappa \left( 1 - \frac{2m_0c^2}{h\nu} \right). \quad (43)$$

Since calculation of the capture of secondary radiation is difficult and differs for each chamber and since it is often small or cancels out of ratios,  $m\mu_{en}$  calculated according to (43) will be used in the analysis of most experiments studied below regardless of how the absorption corrections were made.

In most of the papers analyzed below the author does not give the values of the energy-absorption coefficient he used so, to compare results on a standard basis, new coefficients were calculated. The values of  $m\tau$  and  $m\kappa$  were taken from Grodstein (1957). The values of  $m\sigma_a$  were taken from Lea (1946). The values of  $f$  were taken from Broyles, Thomas, and Haynes (1953). The values of  $h\nu_{\kappa}$  were taken from Hill, Church, and Mihelich (1952). A comprehensive tabulation of energy absorption coefficients based on the latest experimental data has been given by R.T. Berger (1961). Results computed according to (43) are given in table 12.

#### b. Experimental Results

Many early papers comparing two materials exist that have not been covered in this report. They were summarized by Sievert (1940).

A correction common to all these experiments is the amount of absorption in the chamber walls. This varied from less than 1 percent to as much as 86 percent. Each author corrected his own data, but the methods used do not necessarily agree with those of other experimenters. Due to the complexity of the corrections, they have not been recalculated here and there may be a lack of uniformity resulting.

Much of the early work in the study of cavity chambers was done with radium. Radium has a spectrum of gamma ray energies that makes it difficult to analyze results. Most of the gamma rays have energies in the range where Compton scattering predominates in the light elements. In this range of energies and for these elements the ratio of the energy-absorption coefficients in (38) equals the ratio of the number of electrons per unit mass; i.e., it is independent of energy and therefore independent of the choice of the average energy. For the heavy elements studied, particularly lead, photoelectric absorption is appreciable and the value of the

TABLE 12. Mass energy-absorption coefficients

Isotope	Cr <sup>51</sup>	In <sup>113</sup> *	Au <sup>198</sup>	Cs <sup>137</sup>	Ru <sup>106</sup>	Ra <sup>a</sup>	Zn <sup>65</sup>	Co <sup>60</sup> b	Na <sup>24</sup>
1 H.....	0.32	0.336	0.41	0.66	0.726	0.95	1.11	1.25	1.38
4 He.....									
6 C.....	0.0290	0.0291	0.0286	0.0291	0.0291	0.0282	0.0274	0.0268	0.0261
12 Mg.....									
13 Al.....			.0289	0.0284		.0276		.0238	
16 S.....						.0272			
26 Fe.....						.0282			
29 Cu.....			.0313	0.0282		.0283		.0247	
47 Ag.....				0.0324		.0274		.0250	
50 Sn.....			.0404	0.0324		.0273		.0248	
73 Ta.....			.102	0.0484				.0245	
82 Pb.....	.202	0.183	.126	0.0586	.0524	.0406	.0359	.0328	.0307
									2.76

\* Assuming  $\lambda_{\text{eff}} = 0.013 \text{ A}$ .

b Averaging the two energies actually present, 1.17 and 1.33 Mev.

energy-absorption coefficient depends strongly on the energy of the gamma rays. Because of the complexity of the radium spectrum and of the effect of filtration on the spectrum, it was decided to compute the energy-absorption coefficients for use in (38) for the average energy ray of the spectrum and accept the error that this produces in the stopping powers for the heavy elements. Graphite was chosen as the reference substance.

Gray (1936) used small thimble chambers (0.4 cm ID and 1 cm long including a hemispherical end) to compare walls of different atomic numbers for radium gamma rays. A common graphite center electrode was used in all the chambers except the lead chamber which had a lead electrode. The wall thicknesses were chosen to have the same number of electrons/cm<sup>2</sup> as 0.3 cm of graphite except for beryllium which had the equivalent of 0.25 cm; these, and particularly the latter, may have been too thin to give electronic equilibrium. The insulator accounted for about 8 percent of the area of the inner surface of the chamber. Later Gray (1937) used larger chambers (1.2 cm ID) with thicker walls (electronically equivalent to 0.4 cm graphite) and collecting electrodes made of the same material as the walls. Aluminum was the highest atomic number materials used in the latter experiments so the effects of the change in size of the chambers on the measured current ratios due to the variation of  $mJ$  with pressure size were small enough that they were not seen.

Mayneord and Roberts (1937) performed an experiment similar to Gray's with cylindrical chambers, 2 by 2 cm. The collecting electrode was elektron metal for all of the chambers. The wall thickness was varied and the wall correction determined by extrapolating the wall absorption curve to zero thickness. The authors felt that there may have been some beta ray contributions for the thinner walls. Since only low atomic number chambers were used, the effect of pressure-size should be small.

Estulin (1951) employed large flat chambers (20 by 20 by 1 cm) to obtain larger ion currents. The gamma rays were incident perpendicular to the large face and he feels that at worst his chamber offered a path length of about 2 cm to traversing electrons. The collecting electrode was a small brass frame for all chambers. When the sidewalls were the same material as those of the flat faces, Estulin obtained results differing from those of Gray. When he substituted sidewalls of graphite so that 10 percent of the inner surface area was a low atomic number material (compared to 8 percent for Gray), he got results in agreement with Gray.

Myers (1953) used cylindrical chambers that were fairly large (2 cm ID, 10 cm long, 1 g/cm<sup>2</sup> thick). The pressure-size effect should influence his results for the higher atomic number materials. The chamber size is comparable to that used by Estulin so these two authors should nearly agree for the high atomic number chambers.

The results of these authors for radium are collected in table 13. The amount of filtration of the radium sources is indicated in the table. The theoretical value shown was calculated from equations (32) and (34) assuming that the gamma ray energy was 0.95 Mev. This neglects any pressure-size variation of  $mJ$ . The agreement is not very good. Considering the assumptions necessary about the average energy and the differences in filtration, chamber size, and in the amount of low atomic number material in the chamber walls, it is probably surprising that the agreement is as good as it is.

The interpretation of the current ratios obtained for different atomic number chambers when exposed to X-rays is very difficult. A basic difficulty is again that the radiation covers a spectrum of energies. The energies are in the region where photoelectric absorption is very strong, so the choice of an average energy for the spectrum is very critical. Absorption of the X-rays in the chamber wall can change the average energy significantly. Another difficulty is that the chamber must be very small if perturbation of the secondary electron flux traversing the chamber is to be negligible and if production of secondary electrons in the gas is to be negligible. If the atomic number of the wall is less than that of the gas, the greater photoelectric absorption in the gas makes the elimination of the secondary production in the gas very difficult (Attix, 1958). For reasons of this sort, the experiments of Clarkson (1941), Aly and Wilson (1949), and Ibrahim and Wilson (1952) do not give useful information for our present purpose of comparing theoretical and experimental stopping powers.

The results of Attix, DeLaVergne, and Ritz (1958) and of Larson (1956) with X-rays have already been discussed in connection with the variation of  $mJ$  with pressure-size (sec. 5.1). They will be discussed again later since both involved a separate measurement of the dose rate with a free-air chamber (sec. 5.4.c).

In recent years strong sources of radioactive isotopes emitting monoenergetic gamma radiation have become available. These are ideally suited for experiments with cavity chambers since there is no difficulty about the average energy.

TABLE 13.  $m^s_{air}$  relative to graphite for radium gamma rays

Filter Z	Author							Theoretical
	Gray, 1936		Gray, 1937	Mayneord, Roberts, 1937	Estulin, 1951		Myers, 1953	
	0.012 cm Ag; 0.079 cm Pb	2 cm Pb	Unspecified	0.1 cm monel	0.5 cm Pb	2.0 cm Pb	0.05 cm Pt	
Wax <sup>a</sup>	1.157			1.16				1.22
4 Be	.884							.91
6 C	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
12 Mg	.932	.929		.946				.94
Elektron <sup>b</sup>			.939	.939				.98
13 Al	.880	.884						.88
16 S	.914				.85	.86	.94	.91
26 Fe					.67	.67		.84
29 Cu	.68	.75			.62	.64	.76	.75
47 Ag							.68	.66
50 Sn					.50	.51		.64
82 Pb	.31	.32			.43	.50	.53	.55

<sup>a</sup> Taken to be CH<sub>2</sub>.

<sup>b</sup> Taken to be Mg, 92.9%; Al, 3.4%; Zn, 3.3%; Cu, 0.2%.

Myers (1953) used the same chambers he had used with radium to compare different atomic number chambers with Co<sup>60</sup>. In an experiment following his radium work, Estulin (1953) used his flat chambers to compare the currents in lead and graphite chambers for a series of gamma rays with energies between 0.32 and 2.76 Mev. The work of Cormack and Johns (1954) with Co<sup>60</sup>, and Greening (1957) and Attix, DeLaVergne, and Ritz (1958) with Au<sup>198</sup>, Cs<sup>137</sup>, and Co<sup>60</sup> was described in section 5.1. Only the last two of these groups observed the pressure-size variation of  $m^s_{air}$ . The results for Au<sup>198</sup>, Cs<sup>137</sup>, and Co<sup>60</sup> are presented in tables 14.a and 14.b. To avoid the complication of the pressure-size variation of  $m^s_{air}$ , the relative  $s$ 's are given for an average chamber dimension of 1 cm at atmospheric pressure. The theoretical values were calculated from (36).

The results of Attix, DeLaVergne, and Ritz for Au<sup>198</sup> and Cs<sup>137</sup> are given even though they did not have side walls of the same material as the main walls. With this reservation on the validity of their data, the results for these two isotopes are in fair agreement among experimenters and with theory.

All results for Co<sup>60</sup> were for completely enclosed chambers. There is pretty good agreement among the different authors for this isotope. There is good agreement between theory and experiment for the low and medium atomic numbers.

For lead the experimental results are significantly higher than the theoretical. This can be understood in the light of the earlier discussion of the perturbation of the cavity traversing flux (sec. 5.1 and fig. 5). Figure 5 indicates that  $m^s_{air}/m^s_{air}^C$  would be about 13 percent higher than the value calculated from the Spencer-Attix theory due to the perturbation. The results for lead in table 14.b are about 8 percent higher. This is satisfactory agreement.

TABLE 14.a.  $m^s_{air}$  relative to graphite for gamma rays

Gamma emitter	Au <sup>198</sup>				Cs <sup>137</sup>		Theoretical
$h\nu$	0.41						
	Estulin, 1953	Greening, 1957	Attix et al., 1958	Theoretical	Greening, 1957	Attix et al., 1958	
6 C	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13 Al			.91	.84		.90	.86
29 Cu		.68	.73	.71	.72	.76	.72
50 Sn		.44	.49	.58	.56	.64	.60
73 Ta		.52		.51	.50		.53
82 Pb	.575	.48	.47	.50	.53	.48	.52

TABLE 14.b.  $m^s_{air}$  relative to graphite for gamma rays

Gamma emitter	Co <sup>60</sup>					Theoretical
$h\nu$	(1.25)					
	Myers, 1953	Estulin, 1953	Cormack and Johns, 1954	Greening, 1957	Attix et al., 1958	
6 C	1.00	1.00	1.00	1.00	1.00	1.00
13 Al	.90		.89		.90	.87
29 Cu	.75		.73	.76	.75	.75
47 Ag	.64		.65			.66
50 Sn				.62	.66	.63
73 Ta				.56		.54
82 Pb	.59	.57	.58	.53	.58	.54

The effects of cavity perturbation are also evident in table 15 which gives Estulin's results for different energy gamma rays. The theoretical values are from (36) for a 1-cm-atmosphere chamber. Part of his experimental results are higher than the theoretical. Ru<sup>105</sup> and Zn<sup>65</sup> are lower; this might be explained as due to an admixture of lower energy gamma rays. There is no evident reason why the Na<sup>24</sup> experimental value should be so low. The data show the effects of the cavity perturbation but are otherwise valueless in a study

of  $s$ -values until a theory for the perturbation has been developed.

TABLE 15.  $m s_{\text{air}}^2$  relative to graphite for gamma rays  
(ESTULIN, 1953)

Gamma emitter	$h\nu$	$m s_{\text{air}}^2 / m s_{\text{C}}^2$	
		Experimental	Theoretical
Cr <sup>51</sup> .....	0.32	0.625	0.50
In <sup>114</sup> .....	.336	.635	.50
Au <sup>198</sup> .....	.412	.575	.50
Rn <sup>195</sup> .....	.726	.400	.51
Zn <sup>65</sup> .....	1.11	.518	.53
Co <sup>60</sup> .....	(1.25)	.573	.54
Na <sup>24</sup> .....	1.38	.438	.56
	2.76		

### 5.3. Chambers With Different Atomic Number Gases

When a given chamber is exposed to identical fluxes of radiation while filled with different gases, the absorbed dose in the walls is always the same and

$$\frac{m s_{\text{gas}}^{\text{wall}}}{m s_{\text{air}}^{\text{wall}}} = \frac{w_{\text{air}} m J_{\text{air}}}{w_{\text{gas}} m J_{\text{gas}}} \quad (44)$$

Since the relative values of  $w$  are quite well known and only relative current measurements are needed, relative  $s$  values of good accuracy should be obtained.

Hersh and Paterna (described by Failla, 1956) introduced a convenient method for obtaining identical fluxes. They prepared thick, uniformly dispersed beta ray sources by mixing the emitter in polystyrene powder and then molding it into blocks that were used as one plate of an extrapolation chamber. Extrapolation was necessary both to satisfy the Bragg-Gray requirements on chamber size and to eliminate the perturbing effect of the gas on the beta ray dose distribution. Because of the symmetry between the two walls of an extrapolation chamber, the same sort of electronic equilibrium exists at the cavity as would exist in a uniform medium of the emitter. It is necessary to average the currents for opposite polarity collecting voltages to cancel out the current contribution of the beta rays. It was not possible to do this for Ni<sup>63</sup> because of the influence of the collecting field on the large number of low energy beta rays.

Hersh and Paterna used relative  $w$  values determined by Gross (1954) and listed in table 16.

This same technique was used by Baily and Brown (1958a, b) to cover a wider range in atomic number. They used spectroscopically pure noble gases and recirculated the helium and neon over charcoal. This is important because  $w$  for the noble gases is sensitive to the presence of impurities. They used relative  $w$  values taken from Jesse and Sadauskis (1957).

The results of these two groups are given in table 16. Calculations such as those in section 4 for gamma rays have not been made for beta rays so there is difficulty in comparing with theory. To obtain the "theoretical" value given in table 16, it was assumed that an equation of the type (34) should fit the data. Then the value of  $a_{\text{polystyrene}}(T)$  was sought that would give the best fit. The data for all beta ray energies were lumped together for this purpose. The result was  $a=0.17$  which compares well with the gamma ray values given in table 8.

The agreement between the experimental and theoretical values is generally within 1 to 2 percent which is about the experimental uncertainty. The difference is slightly more for krypton and carbon dioxide. The chief differences are for helium, neon, and xenon which gave measured values 30, 20, and 13 percent lower than the theoretical ones. These differences may be due to errors in  $w$ . Jesse and Sadauskis (1953) found that very minute amounts of impurities reduced  $w$  for helium and neon by 30 and 20 percent, respectively—the same amount the observed  $s$ 's are low. It seems that in spite of the care exercised by Baily and Brown, contamination of these gases by traces of gas from the plastic electrodes occurred.

### 5.4. Comparison With Other Measurements of Absorbed Dose

If some other method can be found for measuring the absorbed dose,  $E_d$  in the Bragg-Gray formula, then measurements with a cavity chamber permit a determination of the product  $ws$ . For practical purposes this is all that is needed in use of the Bragg-Gray formula. For our present purpose, however, we will adopt a value of  $w$  in order to compare the  $s$ 's obtained this way with theoretical values or other experimental values. The recently determined value of  $w_{\text{air}}=33.7$  ev per ion pair has been assumed (Bay, 1957; Gross, 1957).

TABLE 16.  $\mu_0 S_{\text{polyethylene}}$  relative to air for beta rays

Beta emitter	Author										Theoretical	
	Nijss		S33		Tijss		Hersh		Bally			
	$E_{\text{max}}$	$\frac{\mu_0}{\mu_{\text{air}}}$										
Gas												
Air	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
H <sub>2</sub>												.40
U <sub>6</sub>												.89
Cs137	780	.83	.887	.88	.888	.81	.81	.81	.81	.81	.81	.87
Co60	778	.80	.78	.78	.78	.78	.78	.78	.78	.78	.78	.79
Ca40	820	.685	.684	.684	.684	.684	.684	.684	.684	.684	.684	.70
CH4	1.034	.992	.997	.990	.993	.994	.994	.994	.994	.994	.994	1.00
Nr	914	1.017	1.021	1.011	1.003	1.018	1.021	1.021	1.021	1.021	1.021	1.02
O <sub>2</sub>	968	1.033	1.014	1.011	1.018	1.015	1.015	1.015	1.015	1.015	1.015	0.99
Co <sub>60</sub>												1.03
Ne												1.29
A												1.48
Kr												1.564
Xe												1.600
												1.87

## a. Total Absorption Method

If all the energy emitted as gamma rays by a source is absorbed in a very large medium, such energy must equal the integral absorbed dose in the medium. If the source is a point source emitting energy  $Q$  in the form of gamma rays

$$Q = \int_0^{\infty} E_z 4\pi r^2 dr = 4\pi sw \int_0^{\infty} Jr^2 dr. \quad (45)$$

Gray (1937a) imbedded a radium source in a large mass of aluminum and measured the ionization in an air-filled cavity as a function of distance from the source.  $Q$  was obtained from a calorimetric measurement of the gamma ray energy emitted per gram of radium. Unfortunately this particular measurement suffered from two large uncertainties. One arose from the fact that the ionization measurements were based on a cavity of 1 cm radius in the aluminum, and a correction was made for the extra attenuation the radiation would undergo to reach an infinitely small cavity located at the center of this volume. This correction was calculated from the theoretical energy absorption coefficients for radium gamma rays and amounted to 6.8 percent. An examination of the measured curve of ionization versus distance indicates that the correction might be as high as 10 percent. So there is an uncertainty of about 3 percent in the ionization measurements from this cause alone. There seems to be an uncertainty of comparable size in the calorimetric determination of  $Q$ . One cannot, therefore, deduce a useful value for the stopping power ratio.

## b. Beta Emitters in Chamber Walls

If a beta ray emitter is uniformly dispersed in a uniform medium such as the walls of a cavity chamber, the absorbed dose rate equals the rate of energy emission per gram.

Gray (1949) employed gel-lined ionization chambers where the radioactive material was uniformly distributed through the gel. Correction had to be made for the size of the chamber, the gamma ray contribution to the ionization, absorption in a paraffin film on the gel, and the failure to attain equilibrium thickness. These, coupled with the uncertainties in the disintegration rates of the isotopes employed, make these data unsuitable for stopping power determinations. This experiment requires absolute values for all factors which makes accuracy extremely difficult.

Caswell (1952) put the active material in water solution to form one electrode of a parallel plate extrapolation cham-

ber. The solution depth was greater than the maximum beta ray range. The extrapolated values of the current per unit volume were used to calculate the average energy per disintegration. By accepting the values of the average energy obtained from beta ray spectrum measurements, the relative stopping power of water to air can be calculated from his data. These are given in table 17 where correction has also been made to  $w=33.7$ . For comparison we can calculate a theoretical value of  $m_s$  for beta rays as was done in section 5.3. It will be assumed that  $a_z(T)$  for water is the same as the value found there for polystyrene,  $a=0.17$ . This gives  $m_s^{\text{water}}=1.15$ . Caswell's values are consistently much lower. He did not find  $J$  proportional to the wall separation so that there are uncertainties in his extrapolation procedure which together with uncertainties in the disintegration rates could account for the discrepancy.

TABLE 17.  $m_s^{\text{water}}$  for beta rays

Isotope	Caswell, 1952	Gross et al., 1957
$S^{35}$		1.17
$Ca^{41}$		
$Ti^{48}$	1.09	
$P^{32}$	1.02	
$Y^{90}$	1.07	
	1.07	

Gross, Wingate, and Failla (1957) performed an experiment almost identical with the previous one. The  $S^{35}$  sample used for the water electrode was energy calibrated by means of a microcalorimeter. Most of the difficulties present in the previous work were eliminated here. The energy liberated per gram of sample does not enter since the 33.7 value for  $w$  was taken from Bay, Mann, Seliger, and Wyckoff (1957) and is based on the same solution. Any basic error in either measurement, however, would certainly influence the result. Using the authors' values, the uncertainty should be less than 2 percent. This time the results were used to find  $w$ . If  $w=33.7$  is used,  $m_s^{\text{water}}$  can be calculated and is given in table 17. It is within the experimental uncertainty of the theoretical value.

### c. Free-Air Chamber

Under electronic equilibrium conditions, the energy absorbed per unit mass in air and in air-equivalent material are the same for a given flux of radiation. This energy is given by  $w_m J_{fac}$  where  $m J_{fac}$  is the current per unit mass of air

measured with a free-air ion chamber. In a material other than air equivalent, the energy absorption will be greater by the ratio of the mass energy-absorption coefficients. From another point of view, the free-air chamber can be considered an air wall chamber in the type of measurements discussed in section 5.2. Equation (38) applies with  $m_s^{\text{air}}$  strictly equal to unity. There is no chance for polarization effects to make the stopping power of the gas and wall different in the case of the free-air chamber. This method has the advantage that the results are independent of  $w$ .

Larson (1956) compared a free-air chamber and aluminum and copper extrapolation chambers for  $K$  fluorescence X-rays of 8.2, 16.1, 23.7, and 34.3 keV. He used computed values of  $m_s$  to predict the relative currents in order to study the applicability of the Bragg-Gray principle in this low energy region. For the case of the copper chamber, capture of fluorescence radiation is appreciable. Fortunately, Larson's chambers are sufficiently close to the example considered in section 5.3.a that that calculation can be used to correct  $m\mu_{en}$ . His copper chamber walls were 0.0013 g/cm<sup>2</sup> thick;  $\tau_K$  is about 40 for copper;  $\tau_{Kt}$  is 0.052 and (41.a) gives  $g=0.82$ . Table 18 shows  $m\mu_{en}$  calculated according to (43) and also corrected for escape according to (40.a) and the corresponding  $m_s$  derived from Larson's data. For comparison, theoretical values calculated according to (32) and (33) are given. For aluminum the agreement between theory and experiment is not quite as good as the 4 to 5 percent accuracy expected for the experiment. There appears to be a systematic trend in the results in a direction contrary to that predicted. In the case of the uncorrected copper data, the agreement is somewhat worse and, in general, applying the escape correction makes it worse still. The large uncertainties in Larson's experiment are associated with the low energies used. At higher energies the method would probably give a better check on theory.

Attix, DeLaVergne, and Ritz (1958) reported similar experiments with heavily filtered X-rays in the range of effective energies from 38 to 206 keV for carbon, aluminum, copper, tin, and lead extrapolation chambers. Mention was made of this work in section 5.1. The ratio  $mJ_z/mJ_{fac}$  was found to vary considerably and nonlinearly with wall separation. This makes the choice of the value for use in (38) arbitrary. Larson (1956) compared his results with those of these authors and concluded that if they had been able to go to wall separations smaller by an order of magnitude they would have obtained current ratios with a small linear variation that could be extrapolated to zero wall separation.

TABLE 18.  $m^s Z_{air}^Z$  for nearly monoenergetic X-rays (Larson 1956)

Z	h $\nu$ kev	$m\mu_{en}^{air}$	With escape correction		Without escape correction		Theoretical $m^s Z_{air}^Z$
			$m\mu_{en}^Z$	$m^s Z_{air}^Z$	$m\mu_{en}^Z$	$m^s Z_{air}^Z$	
13 Al.....	8.2	8.75	47.2	0.84	-----	-----	0.78
	16.1	1.04	6.18	.84	-----	-----	.81
	23.7	.306	1.87	.80	-----	-----	.82
	34.3	.094	.57	.78	-----	-----	.83
29 Cu.....	16.1	1.04	48.4	.69	42.2	0.75	.59
	23.7	.306	18.2	.70	19.1	.73	.63
	22.7	.306	18.2	.70	19.1	.73	.63
	24.3	.094	5.90	.59	6.16	.62	.66

d. Whyte's Method

Whyte (1957) introduced another method for obtaining  $s$  that makes use of the currents in two different chambers and equation (38) but has  $m^s Z_a^Z$  equal to unity.

A corollary to the Fano theorem (sec. 2.4) is that  $mJ$  will be independent of pressure in a cavity chamber if the gas and walls are identical in elemental composition and there is no difference in polarization effect between the walls and gas. For such a chamber,  $m^s=1$ . For Whyte's application it is also necessary to establish that these are the only conditions under which  $mJ$  will be independent of pressure. It is easily seen from the arguments used to establish the theorem that if the elemental compositions are the same, then there must be no difference in polarization effect between wall and gas, and vice versa. This leaves the possibility that both might be different in the wall and gas in such a way to leave  $mJ$  independent of pressure. In terms of the two region model of section 2.4, this requires varying the elemental composition on the two sides of the boundary so that an  $f$ -fold change in source intensity is accompanied by an  $f$ -fold change in stopping power. Since the polarization effect is a function of the electron energy while the elemental composition is not, this can only be done for a single energy, not for the whole spectrum of secondary electrons.

Whyte's procedure is to measure  $mJ$  as a function of pressure in several chambers with walls of different materials but filled with the same gas. Some parameter that measures the variation of the  $mJ$  with pressure is plotted versus the  $mJ$  for a given pressure. It is then assumed that these points are part of a smooth curve and that the value of  $mJ$  on the curve at the point where the parameter indicates zero pressure variation is what would be obtained for a chamber with walls and gas having the same composition and polarization

effect (and, hence,  $m^s=1$ ). Mathematically, the assumption is that both  $mJ$  and its pressure variation are functions of a single variable. In the absence of the polarization effect and in the energy region where Compton effect is practically the only gamma ray interaction (and neglecting our ignorance of cavity perturbation effects), this would be expected to be the case. The source intensities of secondary electrons would depend on the electron densities and their stopping powers would depend on the electron densities and the mean excitation potentials. As in the Fano theorem, we could expect the mutual dependence on electron density to cancel out, leaving the mean excitation potentials of the wall materials as the single variable desired. The inclusion of polarization effects, however, introduces other variables. First, we note that though  $mJ$  theoretically depends on the polarization effect, the pressure variation does not.<sup>17</sup> In Sternheimer's method of calculating density effects, the variables are the mean excitation potentials and the energy levels of the atoms. These are related, of course, and it may be that they can be considered a single variable. In view of the good results obtained by Whyte, this would seem to be the case.

Whyte measured  $mJ$  and its pressure variation for Co<sup>60</sup> gamma rays and chambers of beryllium, graphite, aluminum, and copper. In the pressure-size range he used, about 0.5 to 5 cm-atmospheres,  $mJ$  varied linearly with pressure. The parameter chosen to measure the pressure variation was just the slopes of these lines. The results for  $m^s$  for a 1-cm-atmosphere chamber are given in table 19, together with theoretical values calculated from equation (36). The agreement is excellent except for copper where there is 4.5 percent difference.

TABLE 19.  $m^s Z_{air}^Z$  by Whyte's method for Co<sup>60</sup> gamma rays  
WHYTE (1957)

Z	$m^s Z_{air}^Z$	
	Theoretical	Experimental
4 Be.....	0.900	0.915±0.007
6 C.....	1.003	1.004±0.005
13 Al.....	.874	.877±0.006
29 Cu.....	.747	.71 ±0.01

<sup>17</sup> See equation (37). The pressure variation does not depend on  $\delta$ , but it does depend on the constant part of the polarization effect that is incorporated in the mean excitation potential.

### e. Calorimetric Methods

Calorimetric techniques have had considerable development in recent years and can be applied in several ways to cavity chamber studies. Genna and Laughlin (1955) measured the intensity of a beam of  $\text{Co}^{60}$  gamma rays calorimetrically and then measured the ionization produced by the same beam in a polystyrene extrapolation chamber. The absorbed dose rate in the walls of the chamber is the product of the intensity and the mass energy-absorption coefficient. If the energy-absorption coefficient of the radiation in polystyrene is assumed to be accurately known, the measurements yield the product  $w_m s_{\text{air}}^{\text{polystyrene}} = 39.1 \pm 0.4$  ev/ion pair. If  $w = 33.7 \pm 0.3$  ev/ion pair, this gives  $m_s^{\text{polystyrene}} = 1.16 \pm 0.02$ . The value calculated from equation (36) is 1.12 which disagrees by more than the stated uncertainty.

McElhinney, Zendle, and Domen (1957) did a similar experiment with a beam of 1,400-kilovolt X-rays. In this case the extrapolation chamber had walls of graphite. The energy-absorption coefficients were obtained from Grodstein (1957) and averaged over an empirical X-ray spectrum. From the results one can calculate  $w_m s_{\text{air}}^{\text{graphite}} = 34.0$ . Putting  $w = 33.7$  gives  $m_s^{\text{graphite}} = 1.01$ . Equation (36) gives a value of 1.00 which is probably within the experimental uncertainty.

Myers (1958) measured the total rate of energy emission in the form of beta and gamma rays from a 2-curie  $\text{Co}^{60}$  source enclosed in lead calorimeter. From this and the known decay scheme of  $\text{Co}^{60}$ , the intensity at any point can be calculated. The ionization in an air-filled aluminum cavity chamber (volume 1.33  $\text{cm}^3$ ) at a known distance from the source was measured. His results give  $w_m s_{\text{air}}^{\text{Al}} = 29.0 \pm 0.7$ . For  $w = 33.7 \pm 0.3$ , this gives  $m_s^{\text{Al}} = 0.860 \pm 0.027$ . This is within the experimental uncertainty of the value 0.87 from equation (36).

Bernier, Skarsgard, Cormack, and Johns (1956) made use of calorimetric techniques in an even more direct way. They measured the energy absorbed per gram by graphite and aluminum blocks in a beam of  $\text{Co}^{60}$  gamma rays by calorimetry and then measured the ionization per unit mass of air in 1  $\text{cm}^3$  cavities in graphite and aluminum at the same point. These authors were unable to achieve complete voltage saturation in their chambers. Lack of complete correction would make their results for  $m_s$  too high. The result for graphite was  $mE/mJ = 0.1103 \pm 0.0007$  erg/g per esu/g, and for aluminum  $0.0979 \pm 0.0006$  erg/g per esu/g. These figures give  $w_m s_{\text{air}}^{\text{graphite}} = 33.1 \pm 0.2$  ev/ion pair and  $w_m s_{\text{air}}^{\text{Al}} = 29.4 \pm 0.2$  ev/ion pair. For  $w = 33.7 \pm 0.3$  these in

turn give  $m_s^{\text{graphite}} = 0.982 \pm 0.015$  and  $m_s^{\text{Al}} = 0.873 \pm 0.016$ . For comparison, equation (36) gives 1.00 and 0.87. This is good agreement for the aluminum, but slightly outside the limits of uncertainty for graphite.

### f. Chemical Dosimetry

Chemical dosimeters such as the ferrous sulfate dosimeter can be used to measure absorbed dose. If  $mY$  is the number of ferric ions formed per gram of solution and  $G$  is the number of ferric ions formed per 100 ev absorbed, then  $100 mY/G$  is the energy per gram absorbed by the dosimeter. Sinclair and Shalek (1958) give  $G = 15.6$ . As remarked earlier, cavity theories will apply to chemical dosimeters provided they are thin enough to satisfy the Bragg-Gray requirements. If, instead, the dosimeters are quite thick,  $100 mY/G$  is the average absorbed dose in the dosimeter. The ratio of the absorbed dose in the walls to that in the dosimeter will equal the ratio of the mass energy-absorption coefficients.

Weiss and Bernstein (1955) measured 2-Mv X-rays with a ferrous sulfate dosimeter in a polystyrene container and with a polystyrene extrapolation chamber. Their chemical dosimeter was 0.79 cm thick; an extrapolation was performed to correct for X-ray absorption and change in solid angles, but the smallest thickness actually measured was 0.79 cm. This thickness is greater than the range of practically all of the secondary electrons. The authors assume that the dosimeter is very thick. Then the dosimeter measures the average absorbed dose within itself. The absorbed dose in the polystyrene wall differs from this by the ratio of the energy-absorption coefficients. Then

$$\frac{m_s^{\text{polystyrene}}}{m_s^{\text{gas}}} = \frac{m\mu_{\text{es}}^{\text{polystyrene}}}{m\mu_{\text{en}}^{\text{dosimeter}}} \frac{100 mY}{wG_m J} \quad (46)$$

This approximation is not perfect because the range of many of the electrons is comparable to the size of the dosimeter so that transition effects will occur through a good share of the dosimeter; however, the effects should be small.

Weiss and Bernstein used their data with calculated stopping powers to determine  $w$  values. If we assume  $w_{\text{air}} = 33.7$ , assume the relative  $w$  values found by Gross (table 16) which are independent of any stopping power values, and assume  $w = 26.4$  for argon from Jesse and Sadauskis (1955), the data may be used to calculate  $m_s^{\text{polystyrene}}$ . The results are given in table 20. Theoretical values of  $s$  averaged over X-ray spectra are not available, but the

TABLE 20.  $m_s^{\text{polystyrene}}$  for 2-Mv X-rays

Gas	$m_s^{\text{polystyrene}}$	
	Absolute value	Relative to air
Air.....	1.12	1.00
N <sub>2</sub> .....	1.10	1.00
O <sub>2</sub> .....	1.12	1.02
CO <sub>2</sub> .....	1.12	.99
Ar.....	1.28	1.29
CH <sub>4</sub> .....	.79	.70
C <sub>2</sub> H <sub>4</sub> .....	.92	.79

present results may be compared with those for beta rays in table 16. For this purpose the third column of table 20 shows the present results relative to air. The comparison shows agreement within the 1 to 2 percent experimental uncertainty except for the case of argon. For a comparison of absolute values we can assume that the average energy of the 2-Mv X-rays is about the same as the energy of Co<sup>60</sup> gamma rays. From equation (36)  $m_s^{\text{polystyrene}} = 1.12$  for Co<sup>60</sup>. This is the same as found experimentally for the X-rays.

Zsula, Luizzi, and Laughlin (1957) measured 10- and 20-Mev electrons from a betatron in the same manner. At the depth of the cavity the mean electron energies were 6.3 and 16 Mev. The chemical dosimeter was 0.3 cm thick which is thin enough to permit application of cavity theory; however, the spectrum of electrons at the cavity would be quite different from that for exposure to gamma rays of comparable energies. The authors were able to show that the dosimeter and extrapolation chamber responses were in agreement, using calculated stopping powers, provided the proper correction for the polarization effect were made.

## 6. Conclusions

### 6.1. Practical Applications

In the introduction it was said that the original Bragg-Gray theory, which was based on the assumption of an energy independent ratio of continuous electron stopping powers for the wall and gas, could be considered a first approximation in cavity theory. Laurence's work took into account the energy dependence and could be considered a second approximation. Comparison of the Laurence approximation with

experimental data and with calculations for a third approximation (Spencer-Attix) shows that it is probably in error by no more than 20 percent in the very worst case, that of a lead-wall chamber filled with air. Such a wide difference in atomic number of the wall and gas is hardly ever needed in the practical applications of cavity chambers. On the contrary, the most common use (measurement of exposure dose) requires an air-filled chamber with air-equivalent wall; i.e., no difference in atomic number. Most of the experimental data for walls of low atomic number agree with the Laurence approximation to within the experimental uncertainties, which are generally 1 to 2 percent. Whyte's method (sec. 5.4.d), though subject to some criticism, gives the best available check on the theory for an air-filled cavity in nearly air-equivalent (graphite) walls. The predictions of the Laurence and Spencer-Attix approximations differ by only 0.15 percent for this case, and Whyte's result is within 0.1 percent of both, although only 0.5 percent experimental uncertainty is claimed.

For the practical application of cavity chambers whose walls and gas differ only slightly in atomic number, we recommend that the Bragg-Gray-Laurence theory for  $m_s$  be used.  $m_s$  can be calculated from equations (32) to (34) with the help of the accompanying tables. Most of the mean excitation potentials required can be obtained from tables 3 and 5. Selection of  $I$  values for elements not listed in these two tables can be made on the basis of the discussion in section 3.3.d. For beta rays, the empirical information in section 5.3 may be used in making these calculations.

### 6.2. Cavity Chamber Theory

Although the practical importance of the higher approximations in cavity theory is not very great, they are very important in developing our understanding of the interaction and penetration of radiation through matter. The experiments with chambers having walls of atomic number much different from those of the gas are important because the largest deviations from the lower approximations are to be expected in this case.

The Spencer-Attix theory is the only higher approximation that has been developed to the point where comparison with experiment is generally possible. Their theory is not a rigorous one. At the risk of oversimplifying the physical picture, rigor was reduced to the point where numerical calculations became feasible. A major omission from this (and other) theories is allowance for the effect of the cavity

in perturbing the flux of electrons traversing it. Their theory also does not predict what  $f_z(T, \Delta) = 1/m^s$  will do as  $\Delta$  approaches zero; i.e., pressure-size of the cavity approaches zero.

Spencer and Attix's calculated values of  $f_z(T, \Delta)$  tend to deviate more and more from those calculated in the Laurence approximation as  $\Delta$  becomes smaller. There is no certainty as to what the behavior of  $f_z(T, \Delta)$  should be expected to be as  $\Delta$  approaches zero. Some people feel that as the cavity size becomes negligible, the delta ray effect would vanish, leaving  $f_z(T, \Delta)$  equal to the value calculated in the Laurence approximation. Others see no need for the delta ray effect to vanish. They point out that the energy deposition per gram of gas in the chamber as described by the Bragg-Gray equation is different from what it would be in the same mass of wall material and that this difference does not vanish with chamber size; perhaps the delta ray effect behaves in a similar fashion. The behavior for decreasing  $\Delta$  is of concern to more than just cavity theory because it is involved in the operation of extrapolation chambers. The available experimental data are of no help on this problem because they do not exist for chambers of sufficiently small pressure-size. The Greening (1954) effect prevents the extension of measurements to very much smaller pressure-sizes.

Although theory is lacking for the cavity perturbation and the small-cavity delta ray effects, the following procedure for determining absorbed dose appears to be applicable: measure  $mJ$  as a function of pressure-size of the cavity; multiply  $mJ$  by  $w/f(T, \Delta)$ , where  $f(T, \Delta)$  is obtained from the Spencer-Attix theory, equations (35) and (36). This corrects for the delta ray effect. For not too small pressure-size, the resulting data generally appear to lie on a straight line that can be extrapolated to zero pressure-size. This extrapolation takes care of the cavity perturbation effect. The extrapolated value is the absorbed dose in the wall of the chamber.

Calculations of the Spencer-Attix type have not been made for X-ray energies. The experimental evidence appears to indicate that the delta ray correction must be small for X-rays.

No experimental data have been reported that show evidence of a delta ray effect in beta ray measurements. In particular, the measurements of Baily and Brown (see sec. 5.3) for beta emitters in the plastic walls of chambers containing xenon gas give constant  $mJ$  for small pressure-size. One feels that if a high atomic number wall and low atomic

number gas give a detectable delta ray effect, then there should be a detectable effect for low atomic number wall and high atomic number gas. The plastic-xenon chamber may be compared roughly with a tin-air chamber for which the delta ray effect is small but detectable for gamma rays. It may be that there is some difference between the two types of chambers that makes the delta ray effect in the low  $Z$  wall-high  $Z$  gas chamber so small as to be undetectable.

It is evident that more work, both theoretical and experimental, is needed before we can claim to completely understand cavity chambers. Theory is needed for the cavity perturbation. The theory of the delta ray effect needs development for very small cavities, for X-ray energies, for beta rays, and for low  $Z$  wall-high  $Z$  gas chambers. Experimental data are needed on the same problems.

A topic for which both theory and experiment are lacking at the present is the use of cavity chambers when electronic equilibrium does not exist. This was discussed in section 2.6.a where it was shown that the Bragg-Gray equation should still apply but that  $s$  is expected to be different from its value for equilibrium conditions. One expects that it is not radically different. It would be of interest to know just how much different  $s$  becomes for applications such as dosimetry in the transition region of high energy gamma rays or for tissue near bone.

### 6.3. Stopping Powers

The mean excitation potential,  $I$ , is an experimentally adjusted parameter in the stopping power formulas. At the present time there is considerable experimental uncertainty in the  $I$  values. The values given in table 5 were selected on the basis of the review given in section 3. It is possible that the values for the heavy elements are in error by 20 to 30 percent.  $I$ 's for the lighter elements are generally better known. Aside from any practical applications of the stopping power formulas, it is clearly necessary to improve the measurements of the  $I$  values to answer important theoretical questions about the interaction of charged particles with matter. One or more new proton stopping power experiments, particularly ones at a few hundred Mev energy, are needed for this purpose.

For practical dosimetry purposes, one uses equation (34) for  $m^s$  and deals with walls and gases of low atomic number and similar  $I$ . In equation (34) the error in calculating  $m^s$  is 0.1 to 0.2 the cumulative error in the ratio of the  $I$ 's of the wall and gas. The latter could be several percent, so

the error in  $m_s$  could be several tenths of a percent. This is comparable to the experimental uncertainty in the best cavity chamber measurements. The present  $I$  values, therefore, are good enough for most practical purposes. For special purposes it is desirable that the  $I$  values be known more accurately.

For basic investigations into cavity theory, large differences in atomic number between wall and gas are desired. The effects of errors in  $I$  are then more serious in calculating  $m_s$  and errors of as much as 10 percent can result from 20 to 30 percent errors in  $I$ . The development of a more complete cavity theory will depend on having more certain values for  $I$ .

This study group would like to encourage future experimenters doing stopping power studies to include light elements in their work. These are apt to be overlooked because the present theoretical interest is in elements heavy enough that statistical theories can be applied and tested. Such things as graphite, air, and plastics are very important to dosimetry. The stopping power of graphite has been studied frequently, but materials such as air and plastics are avoided because they are mixtures. Those who must work with these materials would welcome definitive stopping power data for air and a few simple plastics.

The group would also like to encourage more work on polarization and chemical binding effects. These effects are at their largest in the light elements and moderate energies encountered in dosimetry.

## 7. References

The notation (Gatlinburg, 1958) refers to information obtained at the National Academy of Sciences-National Research Council conference on the penetration of charged particles through matter held at Gatlinburg, Tenn., September 1958. The authors of the present report take responsibility for reporting the data and views of the speakers at the conference prior to the issuance of the conference proceedings. (See footnote 11a.)

Allison, S. K., and Warshaw, S. D. (1953). Passage of heavy particles through matter, *Revs. Mod. Phys.* **25** 779-817.  
 Aly, S. M., and Wilson, C. W. (1949). Observations on the ionization produced by high voltage radiation in molded ionization chambers with walls of various effective atomic numbers, *Brit. J. Radiol.* **22** 243-254.  
 Aniansson, G. (1955). New method for measuring the alpha particle range and straggling in liquids, *Phys. Rev.* **98** 300-302.

Attix, F. H., and Ritz, V. H. (1957). A determination of the gamma ray emission of radium, *J. Research NBS* **59** 293-305.  
 Attix, F. H., DeLaVerigne, L., and Ritz, V. H. (1958). Cavity ionization as a function of wall material, *J. Research NBS* **60** 235-243.  
 Baily, N. A., and Brown, G. C. (1958a). Relative stopping power in pure gases, *Radiology* **70** 96.  
 Baily, N. A., and Brown, G. C. (1958b). Electron stopping powers relative to air, Second U.N. Intl. Conf. on the Peaceful Uses of Atomic Energy, paper pp. 742.  
 Bakker, C. J., and Segrè, E. (1951). Stopping power and energy loss for ion pair production for 340 Mev protons, *Phys. Rev.* **81** 489-492.  
 Barber, W. C. (1956). Density effect on the ionization in gases by electrons, *Phys. Rev.* **103** 1281-1284.  
 Barkas, W. H., and von Friesen, S. (1961). High velocity range and energy loss measurements in Al, Cu, Pb, U and emulsion, UCRL-8792. Preliminary results in von Friesen, S., and Barkas, W. H., *Bull. Amer. Phys. Soc.* **4**, 369 (1959).  
 Bay, Z., Mann, W. B., Seliger, H. H., and Wyckoff, H. O. (1957). Absolute measurement of  $w_{air}$  for sulfur-35 beta rays, *Radiation Research* **7** 558-569.  
 Berger, R. T. (1961). The X- or  $\gamma$ -ray energy absorption or transfer coefficient, tabulations and discussion, *Radiation Research*. In press.  
 Bernier, J. P., Skarsgard, L. D., Cormack, D. V., and Johns, H. E. (1956). A calorimetric determination of the energy required to produce an ion pair in air for cobalt-60 gamma rays, *Radiation Research* **5** 613-633.  
 Bethe, H. A., and Livingston, M. S. (1937). Nuclear dynamics, experimental, *Rev. Mod. Phys.* **9** 245-390.  
 Bethe, H. A., and Ashkin, J. (1953). In *Experimental Nuclear Physics*, Vol. 1, E. Segrè (ed.), John Wiley & Sons, Inc., New York.  
 Bichsel, H., Mozley, R. F., and Aron, W. A. (1957). Range of 6 to 18 Mev protons in Be, Al, Cu, Ag, and Au, *Phys. Rev.* **105** 1788-1795.  
 Birkhoff, R. D. (1958). The passage of fast electrons through matter, *Handbuch der Physik* **34** 53-138.  
 Bloch, F. (1933). Bremsvermögen von atomen mit mehreren elektronen, *Z. Physik* **81** 363-376.  
 Bloembergen, N., and van Heerden, P. J. (1951). The range and straggling of protons between 35 and 120 Mev, *Phys. Rev.* **83** 561-566.  
 Bohr, A. (1949). Quoted in Madsen, C. B., and Venkateswarlu, P., *Phys. Rev.* **74** 648-649 (1948).  
 Bragg, W. H. (1910). Consequences of the corpuscular hypothesis of the gamma and X-rays, and the ranges of beta rays, *Phil. Mag.* **20** 385-416.  
 Brandt, W. (1956). Stopping power and valence states, *Phys. Rev.* **104** 691-693.  
 Brandt, W. (1958a). Survey of stopping power, *Health Physics* **1** 11-20.  
 Brandt, W. (1958b). Tight-binding corrections of stopping powers, *Phys. Rev.* **112** 1624-26.  
 Brolley, J. R., and Ribe, F. L. (1955). Energy loss by 8.86 Mev deuterons and 4.43 Mev protons, *Phys. Rev.* **98** 1112-1117.  
 Brown, L. M. (1950). Asymptotic expression for the stopping power of K electrons, *Phys. Rev.* **79** 297-303.  
 Broyles, C. D., Thomas, D. A., and Haynes, S. K. (1953). The measurement and interpretation of the K Auger intensities of  $\text{Sn}^{113}$ ,  $\text{Cs}^{137}$ , and  $\text{Au}^{198}$ , *Phys. Rev.* **89** 715-724.  
 Brysk, H. (1954). Spatial distribution of energy dissipation by high energy X-rays, *Phys. Rev.* **96** 419-425.

- Burch, P. R. J. (1955). Cavity ion chamber theory, *Radiation Research* **3** 361-378.
- Burch, P. R. J. (1957). Comment on recent cavity ionization theories, *Radiation Research* **6** 79-84.
- Burkig, V. C., and MacKenzie, K. R. (1957). Stopping power of some metallic elements for 19.8 Mev protons, *Phys. Rev.* **106** 848-851.
- Caldwell, D. O. (1955). Range-energy relation and masses of the new particles, *Phys. Rev.* **100** 291-294.
- Caswell, R. S. (1952). The average energy of beta-ray spectra, *Phys. Rev.* **86** 82-85.
- Clarkson, J. R. (1941). The measurement by ionization methods of real energy absorption from an X-ray beam, *Phil. Mag.* **31** 437-451.
- Cormack, D. V., and Johns, H. E. (1954). The measurement of high-energy radiation intensity, *Radiation Research* **1** 133-157.
- Ellis, R. H., Rossi, H. H., and Failla, G. (1952). Stopping power of polystyrene and acetylene for alpha-particles, *Phys. Rev.* **86** 562-563.
- Ellis, R. H., Rossi, H. H., and Failla, G. (1955). Stopping power of water films, *Phys. Rev.* **97** 1043-1047.
- Estulin, I. V. (1951). A slotted ionization chamber for the measurement of gamma rays, *J. Exp. Theoret. Phys. USSR* **21** 1412-1415.
- Estulin, I. V. (1953). Dependence of the ionization current on the energy of gamma emission, *J. Exp. Theoret. Phys. USSR* **24** 221-228.
- Failla, G. (1937). The measurement of tissue dose in terms of the same unit for all ionizing radiations, *Radiology* **29** 202-215.
- Failla, G. (1956). In *Progress in Nuclear Energy, Series VII, Vol. 1* (Pergamon Press, London).
- Fano, U. (1954). Note on the Bragg-Gray cavity principle for measuring energy dissipation, *Radiation Research* **1** 237-240.
- Federal Works Agency (1940). *Tables of Sine, Cosine and Exponential Integrals, Vol. II*, Works Progress Administration for the City of New York.
- (Gatlinburg, 1958). See the beginning of this list of references and see section 3.
- Finston, R. A., Hubbell, H. H., Stone, W. G., and Birkhoff, R. D. (1959). Measurement of electron flux in irradiated media by AC methods, Oak Ridge report ORNL-2732.
- Genna, S., and Laughlin, J. S. (1955). Absolute calibration of a cobalt-60 gamma ray beam, *Radiology* **65** 394-405.
- Goldstein, H. (1954). Estimates of the effect of fluorescence and annihilation radiation on gamma ray penetration, Nuclear Development Associates report, NDA-15C-31.
- Goldwasser, E. L., Mills, F. E., and Robillard, T. R. (1955). Direct measurement of the effect of polarization on energy loss, *Phys. Rev.* **98** 1763-1765.
- Gray, L. H. (1929). Absorption of penetrating radiation, *Proc. Roy. Soc. (London)* **A122** 647-668.
- Gray, L. H. (1936). Ionization method for the absolute measurement of gamma-ray energy, *Proc. Roy. Soc. (London)* **A156** 578-596.
- Gray, L. H. (1937). Radiation dosimetry, *Brit. J. Radiol.* **10** 721-742.
- Gray, L. H. (1937a). The rate of emission of gamma-ray energy by radium B and radium C, and by thorium B and thorium C, *Proc. Roy. Soc. (London)* **A159** 263-283.
- Gray, L. H. (1944). The ionization method of measuring neutron energy, *Proc. Cambridge Phil. Soc.* **40** 72-102.
- Gray, L. H. (1949). The experimental determination by ionization methods of the rate of emission of beta- and gamma-ray energy by radioactive substances, *Brit. J. Radiol.* **22** 677-697.
- Greening, J. R. (1954). A contribution to the theory of ionization chamber measurements at low pressures, *Brit. J. Radiol.* **27** 163-170.
- Greening, J. R. (1957). An experimental examination of theories of cavity ionization, *Brit. J. Radiol.* **30** 254-262.
- Grodstein, G. W. (1957). Attenuation coefficients from 10 Mev to 100 Mev, NBS Circ. 583, U. S. Government Printing Office.
- Gross, W. (1954). Determination of  $w$  for different gases and the beta ray spectrum of  $S^{35}$ , *Radiation Research* **1** 495.
- Gross, W., Wingate, C., and Failla, G. (1957). Average energy lost by sulfur-35 beta rays per ion pair produced in air, *Radiation Research* **7** 570-580.
- Hill, R. D., Church, E. L., and Mihelich, J. W. (1952). The determination of gamma ray energies from beta ray spectroscopy and a table of X-ray absorption energies, *Rev. Sci. Instr.* **23** 523-528.
- Hubbard, E. L., and MacKenzie, K. R. (1952). The range of 18 Mev protons in aluminum, *Phys. Rev.* **85** 107-111.
- Hudson, A. M. (1957). Energy loss and straggling of 150 Mev electrons in Li, Be, C, and Al, *Phys. Rev.* **105** 1-6.
- Ibrahim, A. A. K., and Wilson, C. W. (1952). The ionization produced by high voltage radiation in an extrapolation chamber with walls of various atomic numbers and its relation to that to be expected theoretically, *Brit. J. Radiol.* **25** 57-68.
- ICRU (1956). Report of the International Commission on Radiological Units and Measurements, NBS Handbook 62, U.S. Government Printing Office.
- Jensen, H. (1937). Eigenschwingungen eines Fermi-gases und Anwendung auf die Bloch'sche Bremsformel für schnelle Teilchen, *Z. Physik* **106** 620-632.
- Jesse, W. P., and Sadauskis, J. (1953). Alpha particle ionization in pure gases and the average energy to make an ion pair, *Phys. Rev.* **90** 1120-1121.
- Jesse, W. P., and Sadauskis, J. (1957). Absolute energy to produce an ion pair by beta particles from  $S^{35}$ , *Phys. Rev.* **107** 766-771.
- Johns, H. E., and Laughlin, J. S. (1956). Interaction of radiation with matter, in *Radiation Dosimetry*, G. J. Hine and G. L. Brownell (ed.), Academic Press, New York, N.Y.
- Kahn, D. (1953). The energy loss of protons in metallic foils and mica, *Phys. Rev.* **90** 503-509.
- Kalil, F., Stone, W. G., Hubbell, H. H., and Birkhoff, R. D. (1959). Stopping power of thin aluminum foils for 12 to 127 kev electrons, Oak Ridge report ORNL-2731.
- Larson, H. V. (1956). An investigation of the Bragg-Gray principle with fluorescent X-rays, *Radiation Research* **5** 558-568.
- Laurence, G. C. (1937). The measurement of extra hard X-rays and gamma rays in roentgens, *Can. J. Research* **A15** 67-78.
- Lea, D. E. (1946). *Actions of Radiations on Living Cells*, Cambridge University Press, London.
- Lindhard, J., and Scharff, N. (1953). Energy loss in matter by fast particles of low charge, *Kgl. Danske Videnskab. Selskab. Mat-fys Medd.* **27**, No. 15.
- Marinelli, L. D. (1953). Radiation dosimetry and protection, *Ann. Rev. Nucl. Sci.* **3** 249-270.
- Mather, R., and Segrè, E. (1951). Range energy relation for 340 Mev protons, *Phys. Rev.* **84** 191-193.
- Mayneord, W. V., and Roberts, J. E. (1937). An attempt at precision measurement of gamma rays, *Brit. J. Radiol.* **10** 365-386.

- McElhinney, J., Zendle, B., and Domen, S. R. (1957). Calorimetric determination of the power in a 1400 kv X-ray beam, *Radiation Research* **6** 40-54.
- Møller, C. (1931). Über den Stoss zweier Teilchen unter Berücksichtigung der Retardation der Kraft, *Z. Physik* **70** 786-795.
- Myers, I. T. (1953). An ionization chamber method for absolute gamma ray dose measurements, Hanford report HW-29732.
- Myers, I. T. (1958). The measurement of the electron energy required to produce an ion pair in various gases, Thesis, Washington State College, Pullman, Washington.
- Nelms, A. T., *Energy Loss and Range of Electrons and Positrons*, NBS Circ. 577, U.S. Government Printing Office, Washington.
- Nelson, D. R., Birkhoff, R. D., Ritchie, R. H., and Hubbell, H. H. (1958). Measurement of electron flux in media bombarded by X-rays, Oak Ridge report ORNL-2521.
- Phelps, J., Huebner, W. F., and Hutchinson, F. (1954). Stopping power of organic foils for 6 Mev alphas, *Phys. Rev.* **95** 441-444.
- Reynolds, H. K., Dunbar, D. N. F., Wentzel, W. A., and Whaling, W. (1953). The stopping cross section of gases for protons, 30-600 kev, *Phys. Rev.* **92** 742-747.
- Roesch, W. C. (1958). Dose for nonelectronic equilibrium conditions, *Radiation Research* **9** 399-410.
- Sachs, D. C., and Richardson, J. R. (1951). The absolute energy loss of 18 Mev protons in various materials, *Phys. Rev.* **83** 834-837.
- Sachs, D. C., and Richardson, J. R. (1953). Mean excitation potentials, *Phys. Rev.* **89** 1163-1164.
- Sievert, R. M. (1940). Determination of ionization in biological objects, *Acta Radiol.* **21** 189-205.
- Simmons, D. H. (1952). The range-energy relation for protons in aluminum, *Proc. Phys. Soc. (London)* **A65** 454-456.
- Sinclair, W. K., and Shalek, R. J. (1958). The oxidation of ferrous sulfate with 200 kv X-rays,  $Co^{60}$  gamma rays, and 22 Mevp X-rays, *Radiology* **70** 92-93.
- Sonett, C. P., and MacKenzie, K. R. (1955). Relative stopping power of various metals for 20 Mev protons, *Phys. Rev.* **100** 734-737.
- Spencer, L. V., and Fano, U. (1954). Energy spectrum resulting from electron slowing down, *Phys. Rev.* **93** 1172-1181.
- Spencer, L. V., and Attix, F. H. (1955). A theory of cavity ionization, *Radiation Research* **3** 239-254.
- Spiers, F. W. (1956). Radiation units and theory of ionization dosimetry, in *Radiation Dosimetry*, G. J. Hine and G. L. Brownell (ed.), Academic Press, New York.
- Sternheimer, R. M. (1952). The density effect for the ionization loss in various materials, *Phys. Rev.* **88** 851-859.
- Sternheimer, R. M. (1954). The density effect for the ionization loss at low energies, *Phys. Rev.* **93** 351.
- Sternheimer, R. M. (1956). Density effect for the ionization loss in various materials, *Phys. Rev.* **103** 511-515.
- Taylor, L. S. (1951). The measurement of X- and gamma radiation over a wide energy range, *Brit. J. Radiol.* **24** 67-81.
- Thompson, T. J. (1952). Effect of chemical structure on stopping powers for high energy protons, Thesis, University of California, Berkeley. Also in University of California Radiation Laboratory report UCRL-1910.
- Uehling, E. A. (1954). Penetration of heavy charged particles in matter, *Ann. Rev. Nuclear Sci.* **4** 315-350.
- Walske, M. C. (1952). The stopping power of K-electrons, *Phys. Rev.* **88** 1283-1289.
- Walske, M. C. (1956). The stopping power of L-electrons, *Phys. Rev.* **101** 940-944.
- Weiss, J., and Bernstein, W. (1955). Energy required to produce one ion pair for several gases, *Phys. Rev.* **98** 1828-1831.
- Westermarck, T. (1954). Influence of chemical state on stopping power of fast charged particles, *Phys. Rev.* **93** 835-836.
- Whyte, G. N. (1954). Density effect in gamma-ray measurements, *Nucleonics* **12** No. 2 18-21.
- Whyte, G. N. (1957). Measurement of the Bragg-Gray stopping power correction, *Radiation Research* **6** 371-379.
- Wilson, C. W. (1954). Observations on the ionization produced by radium gamma rays in air-walled ionization chambers at low gas pressures, *Brit. J. Radiol.* **27** 158-162.
- Wilson, R. R. (1941). Range and ionization measurements on high speed protons, *Phys. Rev.* **60** 749-753.
- Ziemer, P. L., Johnson, R. M., and Birkhoff, R. D. (1959). Measurement of stopping power of copper by calorimetric methods, Oak Ridge report ORNL-2775.
- Zrelow, V. P., and Stoletov, G. D. (1959). Range-Energy relation for 660-Mev protons, *J. Exptl. Theoret. Phys. (USSR)* **36**, 658-668.
- Zsula, J., Luizzi, A., and Laughlin, J. S. (1957). Oxidation of ferrous sulfate by high energy electrons and the influence of the polarization effect, *Radiation Research* **6** 661-665.

Submitted for National Committee on Radiation Protection and Measurements.

Lauriston S. Taylor, *Chairman*.

Washington, July 25, 1960

No.		Price
73	Protection Against Radiations from Sealed Gamma Sources	\$0.35
74	Building Code Requirements for Reinforced Masonry	.15
75	Measurement of Absorbed Dose of Neutrons, and of Mixtures of Neutrons and Gamma Rays	.35
76	X-ray Protection up to Three Million Volts	.25
77	Precision Measurement and Calibration:	
	Volume I—Electricity and Electronics	6.00
	Volume II—Heat and Mechanics	6.75
	Volume III—Optics, Metrology, and Radiation	7.00
78	Report of the International Commission on Radiological Units and Measurements (ICRU) 1959	.65
79	Stopping Powers for Use With Cavity Chambers	.35