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PILING UP OF COUNTS

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The question of piling up of background counts in an ionization chamber arises so often that it seems worthwhile to make an estimate of the probability of such occurences, even though on the basis of a very simple approximation. This approximation is that a count is recorded whenever the total ionization occuring during a resolving time τ is greater than a certain bias energy E_B.

Let N be the number of charged particles produced in the chamber per second. Then $\bar{n} = N\tau$ is the number expected during a resolving time, and the frequency of getting n particles within one resolving time is

$$p_n = N e^{-\bar{n}} \frac{\bar{n}^{n-1}}{(n-1)!} . \qquad (1)$$

To obtain the counting rate (1) must be multiplied by the probability that n particles produce an ionization greater than E_B, and a sum taken over all values of n.

We shall consider two cases. The first is that the energy of the charged particles is uniformly distributed between zero and E_{max}; this would be the situation for monoenergetic neutrons of energy E_{max} falling on a hydrogen-filled chamber. Write $T = \frac{E_B}{E_{max}}$. A count can occur only if there is more than a T-fold coincidence, i.e., $n > T$. For $n > T$, the probability that the ionization produced is greater than E_B is (see appendix)

Instrumentation

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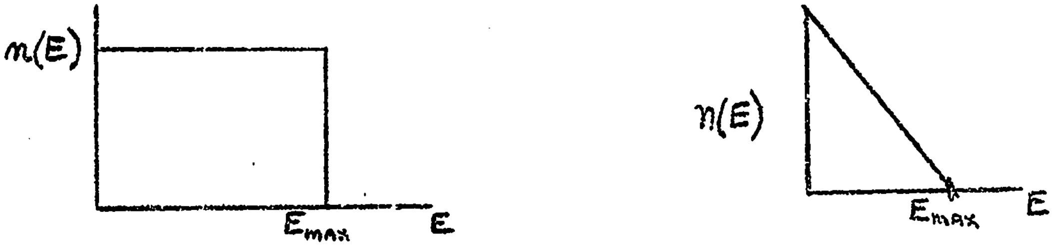
$$F_1(n,T) = \frac{1}{n!}\sum_{r>T}^{n} (-1)^{n-r} C_r^n (r-T)^n,$$

$$C_r^n = \frac{n!}{r!(n-r)!}$$

This function is plotted in Fig. 2.

The counting rate is then $C = \sum_{n>T}^{\infty} p_n F_2(n,T).$



First Case

Second Case

Fig. 1

The second case is that the charged particles have an energy distribution increasing linearly with decreasing energy below E_{max}. This approximately represents the situation for neutrons falling on a paraffin-lined chamber, or for α particles coming from the walls. For this case $F_1(n,T)$ is replaced by

$$F_2(n,T) = \frac{\lambda^n}{(\lambda n)!}\sum_{r>T}^{n} (-1)^{n-r} C_r^n \sum_{S=0}^{n-r} S!\, C_S^{\lambda n} C_S^{n-r} (r-T)^{\lambda n - S},$$

which is plotted in Fig. 3.

The counting rate of course varies very rapidly with bias and resolving time, the dependence being roughly given by

$$C \sim \frac{\bar{n}^T}{(T!)^2}$$

APPENDIX. CALCULATION OF F(n,T)

FIRST CASE

The distribution of charged particle energies is

$$n(t) = 1 \qquad t < 1 \qquad t = \frac{E}{E_{max}}$$

The probability that n particles have an energy greater than E_B is

$$F_1(n,T) = \int_{\substack{\Sigma > T \\ t_i < 1}} dt_1\, dt_2 \cdots dt_n$$

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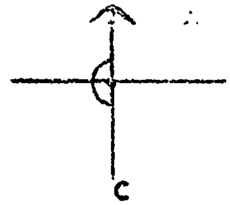
where $\Sigma = \sum^{n} t$

Multiply the integrand by

$$-\frac{1}{2\pi i}\int_c \frac{e^{-K(\Sigma - T)}}{K}\,dK = \begin{cases}1, & \Sigma > T\\ 0, & \Sigma < T.\end{cases}$$



and reverse the order of integration:

$$F_1(n,T) = -\frac{1}{2\pi i}\int_c \frac{e^{KT}}{K}\prod_{i=1}^{n}\int_0^1 e^{-Kt_i}\,dt_i = \frac{1}{2\pi i}\int_c \frac{(1-e^{-K})^n}{K^{n+1}}e^{KT}\,dK$$

$$= \frac{1}{2\pi i}\sum_{r=0}^{n}(-1)^r C_r^n \int e^{-(r-T)K}\frac{dK}{K^{n+1}}$$

$$= \frac{1}{n!}\sum_{r>T}^{n}(-1)^{n-r} C_r^n (r-T)^n.$$

SECOND CASE

Here $n(t) = 2(1 - t)$,

and

$$F_2(n,T) = 2^n \int_{\substack{\Sigma > T\\ t_i < 1}} (1-t_1)(1-t_2)\ldots\ldots(1-t_n)\,dt_1\,dt_2\ldots dt_n$$

$$= -\frac{2^n}{2\pi i}\int_c \frac{e^{KT}}{K}\prod_{i=1}^{n}\int_0^1 (1-t_i)e^{-Kt_i}\,dt_i$$

$$= -\frac{2^n(-1)^n}{2\pi i}\int_c \left[(1-K)-e^{-K}\right]^n e^{KT} K^{-2n-1}\,dK$$

$$= -\frac{2^n}{2\pi i}\sum_{r=0}^{n}(-1)^{n-r} C_r^n \int_c (1-K)^{n-r} e^{-(r-T)K} K^{-2n-1}\,dK$$

$$= \frac{2^n}{(2n)!}\sum_{r>T}^{n}(-1)^{n-r} C_r^n \left[\frac{d^{2n}}{dK^{2n}}(1-K)^{n-r}e^{-(r-T)K}\right]_{K=0}$$

$$= \frac{2^n}{(2n)!}\sum_{r>T}^{n}(-1)^{n-r} C_r^n \sum_{s=0}^{n-r} s!\,C_s^{2n} C_s^{n-r}(r-T)^{2n-s}$$

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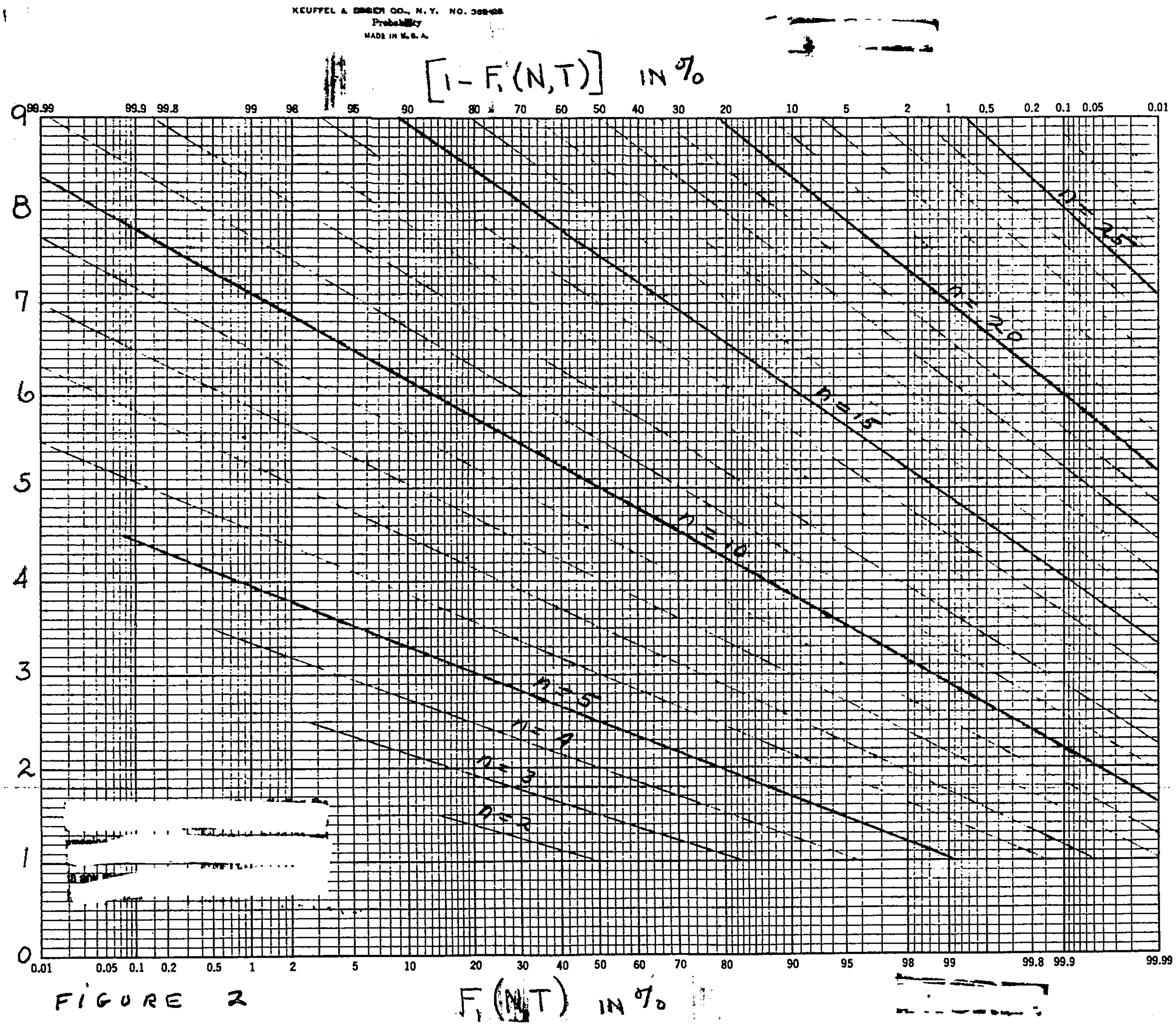
KEUFFEL & ESSER CO., N.Y. NO. 358-23
Probability
MADE IN U.S.A.
[1 - F1(N,T)] IN %
99.99 99.9 99.8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0.5 0.2 0.1 0.05 0.01
T
0 1 2 3 4 5 6 7 8 9
n = 25
n = 20
n = 15
n = 10
n = 5
n = 4
n = 3
n = 2
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
F1(N,T) IN %

FIGURE 2

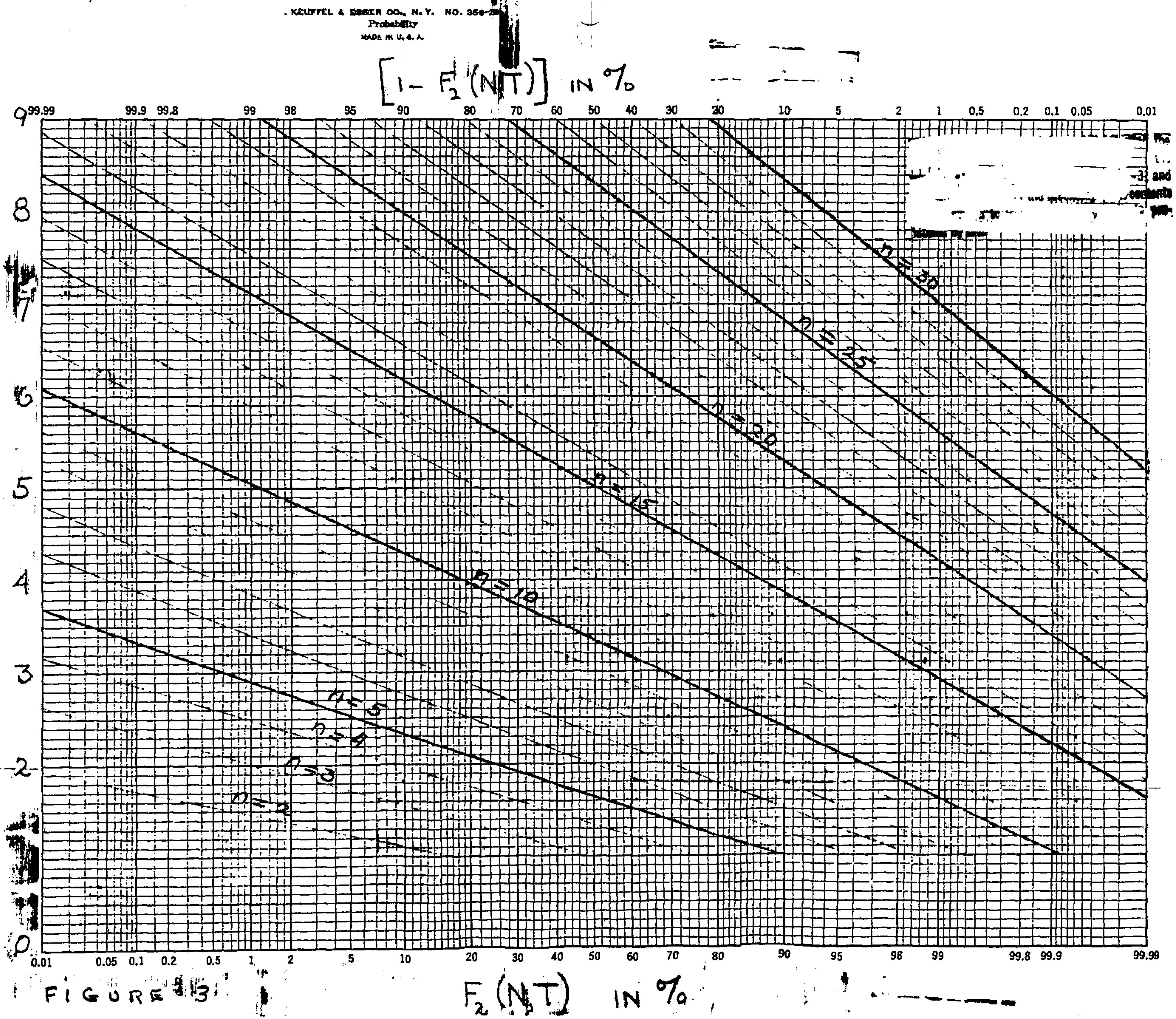

FIGURE 3

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